Model-Free Control Approach for Fixed-Wing UAVs with Uncertain Parameters Analysis

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Abstract—This paper presents first results of an innovative Model-Free Control (MFC) architecture applied to fixed-wing UAVs. MFC is an algorithm dedicated to systems with poor modeling knowledge. Indeed, the costs to derive a reliable and representative aerodynamic model for UAVs motivated the use of such a controller. By exploiting a purely numerical model, this algorithm provides an intuitive method to tune the control loop without any information about the controlled system. We propose to extend the MFC architecture to the case of fixed-wing UAVs and study the MFC properties in terms of uncertain parameters. As a first result, our designed MFC architecture provides a continuous controller able to stabilize the entire flight envelope of two different fixed-wing UAVs. These results show promising adaptive perspectives and demonstrate that MFC presents robust properties for both uncertain parameters and disturbance rejection.

I. INTRODUCTION

The number and diversity of applications involving Micro Air Vehicles (MAVs) are extensive and have received a considerable attention in recent years. Among possible applications, different missions such as aerial imaging [1], atmospheric research [2], or even agricultural tasks [3] require effective performance in terms of endurance, range and high-speed flights which are obtained more efficiently in fixed-wing configurations. These characteristics can be improved for a specific mission profile by using aerodynamic optimization approaches which led to many innovative MAVs [4] [5] [6]. Motivated by the practical problems to find an effective control strategy which is both, simple to transpose for a new MAV and robust in terms of disturbance-rejection remains an interesting challenge for the control community. Therefore, the development of reliable and effective model-based controllers has been an important research topic (e.g., backstepping sliding mode [7], $H_\infty$ controller [8] [9], adaptive control [10] [11], optimal linear controllers [12]). However, these approaches require the development of an accurate model describing the aircraft dynamics that is costly and time consuming. More recently, research works on incremental non-linear dynamic inversion (INDI) [13] have been led and provide a less model-dependent controller that is robust for disturbance rejection. Unfortunately, INDI requires a model of actuators and test flight data to tune the control parameters. For this purpose, Model-Free Control algorithms have been developed providing a potential strategy for designing autopilots without considering any model [14] [15] [16] [17] [18]. Among them, nonlinear MFC strategy [16], has been applied in a nonlinear and strongly coupled system providing good performances in real flights with low computational costs which encourages its use in embedded systems. Whereas MFC approach can be viewed as a potential and efficient method for dealing with identification problems [19] [20]. This control strategy has never been studied on fixed-wing MAVs. While recalling basic motion equations of Fixed-Wing MAVs in §II, the main contributions of this paper are therefore :

• to make explicit in §III the theoretical equations that describe MFC architecture in the benchmarking case of the Fixed-Wing MAVs;
• to provide new preliminary results §IV, focusing on robust properties for both uncertain parameters and disturbance rejection.

II. FIXED-WING MAV MODEL

In order to tackle a wide range of innovative mini-MAVs, various flight dynamics models, in terms of assumptions and numerical techniques, therefore exist. Fixed-wing MAVs are commonly represented by non-linear equations of motion with six-Degrees-of-Freedom (DoF) : 3 DoF correspond to propeller speeds ($\omega_l, \omega_r$) and flap deflections ($\delta_l, \delta_r$).

![Fig. 1: A typical representation of fixed-wing MAV actuators: Propeller speeds ($\omega_l, \omega_r$) and flap deflections ($\delta_l, \delta_r$).](image)
the translation motion \((u, v, w)\) and the 3 remaining DoF to the rotation motion \((\phi, \theta, \psi)\). Based on Newton’s second law with all forces and moments expressed in the body frame, we can describe the MAV dynamics whose angular rates are denoted by \(\Omega = [p \ q \ r]^T\) and their resulting derived equations are given by equation (1) [21].

\[
\begin{align*}
\dot{p} &= \frac{I_{zz}}{I_{xx}} - qr \left( I_{zz} - I_{yy} \right) + qp \frac{I_{zz}}{I_{xx}} + \frac{L^A}{I_{xx}} \\
\dot{q} &= pr \frac{I_{zz}}{I_{xx}} - (p^2 - r^2) \frac{I_{yy}}{I_{xx}} + M^A \frac{I_{yy}}{I_{xx}} \\
\dot{r} &= \frac{I_{zz}}{I_{xx}} \dot{p} - pq \left( I_{yy} - I_{zz} \right) - qr \frac{I_{xx}}{I_{zz}} + \frac{N^A}{I_{zz}}
\end{align*}
\]

Conveniently, the coordinate system was chosen so that the MAV is symmetric in the \(xy\)-plane, thus \(I_{xy} = I_{yx} = I_{zy} = I_{yz} = 0\). And the inertia matrix becomes:

\[
I = \begin{bmatrix}
I_{xx} & 0 & -I_{xz} \\
0 & I_{yy} & 0 \\
-I_{xz} & 0 & I_{zz}
\end{bmatrix}
\]

The resulting translational equations (2) [21], correspond to the linear accelerations.

\[
\begin{align*}
\dot{u} &= (rv - qw) + \frac{X^A}{m} - g \sin \theta + \frac{T_h}{m} \\
\dot{v} &= (pu - ru) + \frac{Y^A}{m} + g \cos \theta \sin \phi \\
\dot{w} &= (qu - pv) + \frac{Z^A}{m} + g \cos \theta \cos \phi
\end{align*}
\]

Where \((u, v, w)\) are the linear velocities expressed in the body frame, \(g\) the gravitational constant and \(\phi, \theta, \psi\) the MAV attitude, respectively, roll, pitch and yaw angles. The thrust of the propellers \((T_h)\) which is a squared function of propeller speeds also depends on the air density \((\rho)\) and propeller characteristic, such as the diameter, etc. Aerodynamic forces \((X^A, Y^A, Z^A)\) and aerodynamic moments \((L^A, M^A, N^A)\) are subject to aerodynamic coefficients:

\[
\begin{bmatrix}
X^A \\
Y^A \\
Z^A
\end{bmatrix} = \frac{1}{2} \rho S V^2 \begin{bmatrix}
C_x \\
C_y \\
C_z
\end{bmatrix}
\]

\[
\begin{bmatrix}
L^A \\
M^A \\
N^A
\end{bmatrix} = \frac{1}{2} \rho S V^2 \begin{bmatrix}
b C_l \\
c C_m \\
b C_n
\end{bmatrix}
\]

where \(S, b, c\) are respectively, the wing area, the wingspan and the mean chord.

Remark : Aerodynamic forces can also be modelled using the \(\Phi\)-Theory proposed by [22].

The kinematic attitude equations (3) are used to relate the angular rates to Euler angles [21].

\[
\begin{align*}
\dot{\phi} &= p + \tan \theta (q \sin \phi + r \cos \phi) \\
\dot{\theta} &= q \cos \phi - r \sin \phi \\
\dot{\psi} &= \sec \theta (q \sin \phi + r \cos \phi)
\end{align*}
\] (3)

The nonlinear state space representation corresponding to the Fixed-Wing MAV can be described in a compact form such as: \(\dot{x} = f(x, u)\) and \(y = h(x, u)\), where \(x = (\omega_x, \omega_y, \omega_z)^T\) and \(u_t, \omega_b \in \mathbb{R}^3, q \in \mathbb{R}^4\), denote respectively, vehicle velocity in local NED frame, angular velocity in body frame, and vehicle attitude represented in quaternion formulation. Control inputs \(u = (\omega_x, \delta_l, \delta_r)^T\) are defined according to Fig. 1.

III. MODEL FREE CONTROL

Model-Free Control term appears many times in the literature, but in distinct meanings from this paper. Actually, the growing importance of artificial intelligence and machine learning techniques, particularly through neural networks, has naturally been implanted into the model-free terms: see, for example [23] [24]. However, in this paper, we assume model-free control terms according to [20].

A. MFC Theory

We present briefly the main theoretical principles of some research works dealing with model-free control approach.

Let’s consider the following non linear state-space representation defined by :

\[
\begin{align*}
\dot{x} &= f(x, u) \\
y &= h(x, u)
\end{align*}
\]

where \(x, u, y\) are the state, input and output vectors respectively. The output \(y\) is not directly available but rather it is observed through a noise corruption. A model for the output vector can be described by the following equation :

\[
y_m = y + \omega_n
\]

where \(\omega_n\) is the vector observation noise of different outputs. The exploitation of the MFC principles required the definition of a particular SISO model, named Ultra-Local Model, which corresponds to replace the unknown dynamic by a purely numerical model :

\[
y_m^{(v)} = F_y + \alpha \cdot u
\]

In equation (6), \(v\) is the order derivative of \(y_m\), \(\alpha\) is a non-physical constant parameter and is an element of \(\mathbb{R}\). Moreover, the exploitation of this numerical model requires the knowledge of \(F_y\). This quantity represents the real dynamics of the model as well as the different disturbances which could damage the output-system performances. Thus, an accurate estimation of \(F\), defined as \(\hat{F}\), is crucial and plays an important role in the MFC performance. Assuming that we do not have any information about the plant, its estimation can be computed directly by considering the following methodology in which we use a second-order Ultra-Local Model :

\[
y_m = F_y + \alpha \cdot u
\]

\(^1\) The same methodology can be applied to find the mathematical expression of \(F_y\) for a first-order Ultra-Local Model.
The first step is to apply the Laplace Transform in the equation (7). Referring to elementary operational calculus we transform the equation (7) to equation (8):

$$s^2Y_m(s) - sy_m(0) - \dot{y}_m(0) = \frac{F_y}{s} + \alpha U(s)$$ (8)

Where $Y_m(s)$ and $U(s)$ correspond to the Laplace transforms of $y_m$ and $u$. By differentiating twice the previous equation we are able to rid the initial condition:

$$2Y_m(s) + 4s \frac{dY_m(s)}{ds} + s^2 \frac{d^2Y(s)}{ds^2} = \frac{2F_y}{s^3} + \frac{\alpha d^2U(s)}{ds^2}$$ (9)

However, $s$ in the time domain corresponds to the derivation with respect to time and it is sensitive to noise corruptions. Therefore, in order to reduce both noise and numerical computation errors on the output estimation, we replace $s$ in the time domain with $\hat{s}$ which have robust properties with respect to noise. Thus, multiplying both sides of equation (9) by $\hat{s}^{-3}$, we obtain:

$$2Y_m(s) + 4 \hat{s} \frac{dY_m(s)}{ds} + \frac{1}{\hat{s}} \frac{d^2Y(s)}{ds^2} = \frac{2F_y}{\hat{s}^3} + \frac{\alpha d^2U(s)}{ds^2}$$ (10)

Using inverse Laplace operator, equation (10) can be transferred back to the time domain employing convolution formula and classic Inverse Laplace transforms or Cauchy’s formula to reduce multiple integrals in a simple one:

$$\hat{F}_y = \frac{5}{2T^5} \int_{t-T}^{t} [(T-\sigma)^2 - 4\sigma(T-\sigma) + \sigma^2] y_m(\sigma) \left[-\frac{\alpha}{2} (T-\sigma)^2 u(\sigma) \right] d\sigma$$ (11)

From measurements of $y_m$ and $u$ the unmodeled dynamic of $y$ and the disturbances $\omega_n$ are estimated by $\hat{F}_y$ which is updated for each interval of integration $[t-T, t]$. This interval corresponds to the window width of a receding horizon strategy which results in a trade-off. The idea is to choose the window width small so as to calculate the estimation within an acceptable short delay but large enough in order to preserve the low-pass filter properties whose noise attenuation of $y_m$. Based on such estimator it is possible to design a robust controller that estimates on-line the system dynamic from periodic measurements of $y_m$ and $u$. The general form of the close-loop control can be defined such as:

$$u = -\frac{\hat{F}_y}{\alpha} + \frac{y_d^{(v)} + K(\xi)}{\alpha}$$ (12)

where the quantity $\xi = y_m - y_d$ is the tracking error and $K(\xi)$ is a closed loop feedback controller. We recognize in equation (12) the typical mathematical expression of a “nominal control” in the “flatness-based” control (see [25] [26] for details) in which the non-linear terms $\hat{F}_y$ is summed with a closed loop tracking of a reference trajectory $t \rightarrow y_d(t)$.

B. Illustrative example

We consider now a simple pitch angle dynamic of a given aircraft, the transfer function between the output ($\theta$) and the elevator control input ($\delta_e$) is described as follows:

$$TF(s) = \frac{\theta(s)}{\delta_e(s)} = \frac{1.151s + 0.1774}{s^3 + 0.739s^2 + 0.921s}$$ (13)

A second order Ultra-Local Model ($\nu=2$) was chosen to estimate the pitch dynamic ($\dot{\theta}$):

$$\ddot{\theta}_m = F_y + \alpha \cdot \delta_e$$ (14)

Figure 2 shows the MFC architecture which the closed-loop control can be computed by :

$$\delta_e = -\hat{F}_\theta + \ddot{\theta}_d + K(\xi_\theta)$$ (15)

Fig. 2: Detailed Model-Free Control schema applied on equation (13). Proportional-Derivative control $K$, $\hat{F}_\theta(t)$ estimator of pitch dynamic and disturbances with $\alpha$ a non-physical constant parameter.

Fig. 3: Dark-Knight MAV.

Fig. 4: Cyclone MAV.
Replacing (15) in (14) with $K$ equal to Proportional-Derivative gains, we have:

$$\ddot{\theta}_m = F_\theta - \dot{F}_\theta + K_p \xi_\theta + K_d \dot{\xi}_\theta$$  \hspace{1cm} (16)

It follows that theoretically, if the error between the pitch angle estimator and the real pitch angle, is approximately zero during $[t - T, t]$:

$$F_\theta - \dot{F}_\theta \approx 0$$

The pitch angle and the dynamic error ($\xi_\theta$) can be easily tuned by proportional and derivative gains, respectively $K_p$ and $K_d$ such that:

$$\ddot{\xi}_\theta = \ddot{\theta}_m - \ddot{\theta}_d$$  \hspace{1cm} (17)

$$\ddot{\xi}_\theta = \dddot{\theta}_m - K_p \xi_\theta - K_d \dot{\xi}_\theta = 0$$  \hspace{1cm} (18)

The MFC performance varies according to the following parameters: the length of the integration window $T$; The coefficient $\alpha$ that is chosen to set the same magnitude between $\theta_m$ and the control input $\delta_c$, $K_p$ and $K_d$ which are used to set the error dynamic, see Fig. 5.

**Remark**: It is important to emphasize that MFC algorithms have been developed to Single-Input Single-Output (SISO) systems and Fixed-Wing MAVs are Multiple-Input Multiple-Output (MIMO) systems. In our study-case, a second order Ultra-Local Model ($\nu=2$) was chosen to represent each state dynamic of the MAV (attitude and velocities). Wherefore, a control architecture composed by multiple SISO MFCs is proposed, and developed in the MFC architecture block, see Fig. 6.

### IV. FLIGHT SIMULATIONS

We now apply the control approach described in the previous section for two fixed-wing MAVs (Fig. 3 and Fig. 4) whose specifications are described in Table I. The idea is to study the MFC properties in terms of uncertain parameters. The simulation is discretized at 500 Hz and includes additional sensor noises and state estimation errors. Also, inspired by the Dryden Wind Turbulence Model, we add wind gusts of around 4 (m/s) along x and y axes to perturb the lateral and longitudinal motions. An overview of the simulation is shown in Fig. 7. The flight path describes a take-off with a constant rate of climbing fixed at 2.5 (m/s), see Fig. 7d. Reaching a desired altitude, the rate of climbing is ordered to zero to maintain the flight level. During this part of the flight, we can analyze the longitudinal dynamics, such as forward speed, rate of climbing and pitch angle. At constant altitude, we imposed left-right trajectories were imposed to validate the roll and yaw control loops. Positive east-velocity defines a positive-desired roll angle and the MAV turns right, see Fig. 7c and Fig. 7e. By analogy, a rate of climb greater than zero calls for positive pitch angles (Fig. 7d and Fig. 7f) and a higher flight level will be reached. The reverse is also true, the MAV can turn left and reaches a smaller flight level with negative-desired velocities. The thrust computed by MFC can be analyzed into two parts: the nominal thrust and the differential thrust. In the first one, both propellers turn at the same speed to ensures a forward velocity around fifteen meters per second, Fig. 7b. In the second one, propellers turn at different speeds creating a moment around the z axis. This moment controls the yaw angle that is set to zero throughout the simulation, as shown in the Fig. 7g. The performance of the actuators are presented in the Fig. 7h and Fig. 7i. Cyclone flaps present greater deflection angle than for Dark-Knight. This difference may be attributed to the fact that the Cyclone has a smaller

![Fig. 5: Pitch angle response for different combinations of $\alpha$ and $T$ with $K_p = -1.5$ and $K_d = -2.5$.](image)

![Fig. 6: MFC architecture designed for HMAsVs with saturated control inputs. Propeller speeds ($\omega_l$, $\omega_r$) and flap deflections ($\delta_l$, $\delta_r$) are computed by means of MFC architecture block.](image)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Cyclone</th>
<th>Dark-Knight</th>
<th>SI Units</th>
<th>$\Delta$ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>0.852</td>
<td>0.586</td>
<td>[Kg]</td>
<td>45.39</td>
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<tr>
<td>$I_{xx}$</td>
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<td>0.00541</td>
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<td>52.30</td>
</tr>
<tr>
<td>$I_{yy}$</td>
<td>0.00798</td>
<td>0.00523</td>
<td>[Kg m$^2$]</td>
<td>52.47</td>
</tr>
<tr>
<td>$I_{zz}$</td>
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<td>0.01082</td>
<td>[Kg m$^2$]</td>
<td>51.20</td>
</tr>
<tr>
<td>Propeller radius</td>
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<td>0.1524</td>
<td>[m]</td>
<td>33.33</td>
</tr>
<tr>
<td>Mean Chord</td>
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<td>0.175</td>
<td>[m]</td>
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<tr>
<td>Wingspan</td>
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<tr>
<td>Wing area</td>
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<td>0.175</td>
<td>[m$^2$]</td>
<td>16.98</td>
</tr>
</tbody>
</table>
Fig. 7: Forward flight simulations. On the top: the flight path. From left to right: forward-speed, east velocity and velocity along z axis. Attitude in the third line and on the bottom: propeller speeds ($\omega_l < 0$ and $\omega_r > 0$) due to counter-rotation sense, elevon deflections, convention negative for pitch-up ($\delta_l$ and $\delta_r$) and wind disturbances.
wingspan. Thus, for an equivalent airspeed, the Cyclone needs a higher pitch angle to generate lift and to reach the desired rate of climbing. The zoom in the Fig. 7i (around 45 seconds), allows us to see the command which generates a negative roll moment that corresponds to a left turn. Despite windy conditions Fig. 7j, MFC ensures effective attitude stabilization and tracking velocities for both MAVs during lateral and longitudinal trajectories.

V. CONCLUSION

We have presented velocity and attitude control using MFC architecture for fixed-wing MAVs. The proposed approach is able to stabilize the entire flight envelope without any knowledge about the controlled MAV. First results demonstrated an effective disturbance rejection and flight simulations showed the effective control of unmodelled dynamics with MFC by the means of its adaptive properties.

This control architecture and MFC algorithms are being implemented in Paparazzi open-source autopilot system (cf. Paparazzi project at: https://wiki.paparazziuav.org/) and experimental flights will be presented soon.

REFERENCES


