FORECASTING MOCK EXAM (2019 EXAM)

1 a) SEE # 100 FOR A SIMILAR QUESTION (REFER TO FIGS. 1 – 3)

1 b) SEE # 101 FOR A SIMILAR QUESTION (REFER TO FIG. 4)

YOU CAN ALSO REFER TO THE LR STATISTIC ($LR = 9.40$ [0.15] DNR H_0)

– SEE ALSO # 106

... AND McFADDEN'S R^2 ($R^2_M = 0.15$)

– SEE ALSO # 108

1 c) SEE # 100 FOR A SIMILAR QUESTION

AT THE MEAN, THE CHARACTERISTICS ARE: (FIG. 1), TO 2 D.P.

$$\begin{array}{ccccccccc} & & \text{CONSTANT} & \text{CRIME} & \text{DEFENSE} & \text{FEDFUNDS} & \text{UN} & \text{S} & \text{W} \\ \bar{x}' & = & (1, & 5221.80, & \underset{0.434920}{0.43}, & 5181.14, & \underset{6.822}{6.82}, & 0.34, & 0.16) \end{array}$$

AND THE ESTIMATED PARAMETERS ARE: (FIG. 4), TO 2 D.P.

$$\begin{array}{ccccccccc} & & \text{CONSTANT} & \text{CRIME} & \text{DEFENSE} & \text{FEDFUNDS} & \text{UN} & \text{S} & \text{W} \\ \hat{\beta}' & = & (\underset{-1.340666}{-1.34}, & \underset{-0.0000511}{-0.000051}, & \underset{-0.467755}{-0.47}, & \underset{0.0000826}{0.000083}, & \underset{0.329023}{0.33}, & \underset{-0.873189}{-0.87}, & \underset{-0.418786}{-0.42}) \end{array}$$

THE MARGINAL EFFECTS (M.E.) FOR THIS PROBIT MODEL ARE:

$$\left.\underset{7\times 1}{\frac{\partial \widehat{Prob}(DEM=1)}{\partial x}}\right|_{x=\bar{x}} = \phi(\bar{x}'\hat{\beta})\hat{\beta} \; ; \quad \phi(u) = \frac{1}{\sqrt{2\pi}} e^{-u^2/2}$$

SO, $\bar{x}'\hat{\beta} = 0.4977310654$
$\simeq 0.50$ (USE FULL ACCURACY THROUGHOUT THE CALCULATION)

AND $\phi(\bar{x}'\hat{\beta}) = 0.35246405275$
$\simeq 0.35$

GIVING MARGINAL EFFECTS

$$\phi(\bar{x}'\hat{\beta})\hat{\beta} \simeq \begin{array}{c} \begin{pmatrix} -0.47 \\ -0.000018 \\ -0.16 \\ 0.000029 \\ 0.12 \\ -0.31 \\ -0.15 \end{pmatrix} \end{array} \begin{array}{l} \text{CONSTANT} \\ \text{CRIME} \\ \text{DEFENSE} \\ \text{FEDFUNDS} \\ \text{UN} \\ \text{S} \\ \text{W} \end{array}$$

E.G. DEFENSE $\uparrow 1 \Rightarrow$ $\widehat{Prob}(DEM=1) \downarrow 0.16$

1 d) SEE # 113 FOR A SIMILAR QUESTION (FIG. 9)

$\widehat{Prob}(DEM=1) \uparrow$ IN UN (UNEMPLOYMENT RATE);

INCREASES MOST RAPIDLY FOR LOW UN

$\widehat{Prob}(DEM=1)$ HIGHEST FOR $(S,W)=(0,0)$, LOWER FOR $W=1$, LOWER STILL FOR $S=1$.

1 e) SEE # 115 FOR A SIMILAR QUESTION (FIG. 10); $\widehat{Prob}(DEM=1) \geq 0.5 \Rightarrow \widehat{DEM} = 1$

		ACTUAL DEM=0	ACTUAL DEM=1	
MODEL	DEM=0	9	5	14
	DEM=1	8	28	36
		17	33	50
		53%	85%	74%

CF. $R^2_M = 0.17$ (FIG. 4)

NAÏVE MODEL (ALL $\widehat{DEM}=1$) CORRECT IN $33/50 = 66\%$ OF CASES, SO OVERALL MODEL PERFORMANCE (74%) IS SUPERIOR; ALTHOUGH MODEL IS MUCH BETTER AT PREDICTING DEM=1 THAN DEM=0.

1 f) SEE # 91 FOR EXPLANATION; AND # 114 FOR MORE DETAIL. (FIG. 6)

WE EXPECT

$$\hat{\beta}_{LOGIT} \simeq \frac{\pi}{\sqrt{3}} \hat{\beta}_{PROBIT} \; ;$$

THIS IS BECAUSE $\Lambda(.)$ IS LEPTOKURTIC RELATIVE TO $\Phi(.)$, AND THE STANDARD LOGISTIC HAS VARIANCE $\pi^2/3$.

1g) SEE # 93 AND # 110 FOR DETAILS AND APPLICATION.

FIG 1, FIG 6

ODDS-RATIO $OR = e^{x'\beta}$ FOR LOGIT MODEL.

FIG 1 (SEE 1c):

$\bar{x}' = (1, 5221.80, 0.434920, 5181.14, 6.822, 0.34, 0.16)$

FIG 6 (CAREFUL! NEED $\hat{\beta}_{LOGIT}$ NOT $\hat{\beta}_{PROBIT}$)

$\hat{\beta}' = (-2.442380, -0.0000802, -0.818580, 0.000148, 0.580741, -1.538007, -0.776971)$

SO $\bar{x}'\hat{\beta} = 0.8642009084 \approx 0.86$

AND $\hat{OR}\Big|_{x=\bar{x}} = 2.373108997 \simeq \underline{2.37}$

1h) SEE # 98 FOR DETAILS, AND

$\ln \mathcal{L}(\beta) = \sum_{i=1}^{n} \{y_i \ln F(x_i'\beta) + (1-y_i)\ln(1-F(x_i'\beta))\}$

< 0

FOR $Prob(y_i=1) = F(x_i'\beta)$; $y_i = 0,1$
AND $x_i'\beta \not\to \pm\infty \Rightarrow F(x_i'\beta) \in (0,1)$.

1i) SEE # 83 (FIGS. 7, 8) AND # 84.

$d = 1.587490$; $H_0: \rho = 0$, $H_1: \backslash H_0$

$\frac{\sqrt{T}}{2}(d-2) \sim N(0,1)$ as $T \to \infty$

$T = 50$; -1.4584

95% ± 1.96

90% ± 1.65

DNR H_0 (NO FIRST-ORDER AUTOCORRELATION) AT 90% LEVEL.

BREUSCH-GODFREY BG $(p=2)$: $\sim \chi^2(2)$

$BG = 5.672227\ [0.0587]$

H_0: NO AUTOCORRELATION UP TO LAG p

H_1: $u_t \sim AR(p)$ OR $u_t \sim MA(p)$

DNR H_0 AT 95%.

R H_0 AT 90%.

BUT BG IS TESTING FOR FIRST OR SECOND-ORDER AUTOCORRELATION, WHILE DURBIN-WATSON d ONLY CONSIDERS FIRST-ORDER AUTOCORRELATION.

2a) FIGS 13 – 16 SUGGEST THAT Y_t HAS A UNIT ROOT (THIS IS NOT A RIGOROUS TEST THOUGH...), AND SO Y_t NEEDS TO BE TRANSFORMED TO STATIONARITY BEFORE APPLYING BOX–JENKINS ARMA MODELLING; SO (FIGS 18 – 20), $Y_t \sim ARMA(1,2)$ IS NOT APPROPRIATE — NOTE THAT THE ESTIMATED AR ROOT IS STILL CLOSE TO THE BOUNDARY OF THE UNIT CIRCLE...

SEE #144 AND #145

2b) FIG 25 $\Delta Y_t \sim AR(1)$ SUGGESTS THAT ΔY_t DOES NOT HAVE A UNIT ROOT (AND IS STATIONARY), SO WE CAN USE BOX–JENKINS. THEN (FIGS. 22 – 30), $\Delta Y_t \sim MA(2)$ MINIMIZES THE SCHWARZ INFORMATION CRITERION (SIC) ACROSS ALL COMBINATIONS OF $p,q = 0,1,2$ AND $\Delta Y_t \sim ARMA(p,q)$. THE MODEL IS INVERTIBLE (FIG 31), STATIONARY (AS ALL MA(q) ARE), FITS THE DEPENDENCE REASONABLY WELL (FIG. 32), HAS WHITE NOISE ERRORS (FIG. 33); AND HAS A REASONABLE DYNAMIC SHORT-TERM FORECAST

FIG. 34 (ALTHOUGH WITH A WIDE CONFIDENCE INTERVAL)

SEE #144 AND #145

AND $R^2_M = 0.12$

2c) FROM #140,
AND 2d)

WITHOUT LOSS OF GENERALITY, CAN IGNORE α_0 WHEN COMPUTING $\rho_s \cdots$

$\Delta y_t \sim MA(1) \Rightarrow \Delta y_t = \varepsilon_t + \beta_1 \varepsilon_{t-1}$; $\varepsilon_t \sim IID(0, \sigma^2_\varepsilon)$

AND $\rho_s = \begin{cases} 1 & ; s=0 \\ \beta_1/1+\beta_1^2 & ; s=1 \\ 0 & ; s=2,3,4\ldots \end{cases}$

THEN (FIG. 23). $\hat{\beta}_1 = 0.403445$

$\Rightarrow \underline{\rho_1 \approx 0.3470 \approx 0.3}$

FROM CLASS THEORY SOLUTIONS,

$\Delta y_t \sim MA(2) \Rightarrow$

$\Delta y_t = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2}$; $\varepsilon_t \sim IID(0, \sigma^2_\varepsilon)$

THEN (FIG. 24) $\hat{\beta}_1 = 0.293572$

$\hat{\beta}_2 = -0.212582$

AND $\rho_s = \begin{cases} 1 & ; s=0 \\ \dfrac{\beta_1(1+\beta_2)}{1+\beta_1^2+\beta_2^2} & ; s=1 \\ \dfrac{\beta_2}{1+\beta_1^2+\beta_2^2} & ; s=2 \\ 0 & ; s=3,4,\ldots \end{cases}$

$\Rightarrow \underline{\rho_1 \approx 0.2043 \approx 0.2}$

$\underline{\rho_2 \approx -0.1879 \approx -0.2}$

2e) $y_t = y_{t-1} + \varepsilon_t$; $t=1,2,\ldots,T$;

$\varepsilon_t \sim IID(0, \sigma^2_\varepsilon)$; y_0 INITIAL

THIS IS A DRIFTLESS RANDOM WALK (#130) $\subset$ AR(1).

SOLVING BACKWARDS (SINCE $A(L) = 1-L$ HAS NO INVERSE)

GIVES (#133)

$$y_t = \alpha_0 t + y_0 + \sum_{i=0}^{t-1} \varepsilon_{t-i} ;$$

DRIFT $\alpha_0 = 0$ HERE, SO

$$y_t = y_0 + \sum_{i=0}^{t-1} \varepsilon_{t-i}$$

AND $\underline{E(y_t)} = y_0 + \sum_{i=0}^{t-1} E(\varepsilon_{t-i})$

$= \underline{y_0 < \infty}$

THIS IS NOT EVIDENCE OF NONSTATIONARITY, SINCE THE MEAN IS FINITE. IT WILL BE SUFFICIENT TO SHOW THAT $\underline{E(y_t^2) \to \infty}$ SINCE

$Var(y_t) = E(y_t^2) - (E(y_t))^2$

WILL THEN EXPLODE, WHICH INVALIDATES STATIONARITY (SINCE SOME MOMENTS OF y_t ARE NOT FINITE).

$y_t = y_0 + \varepsilon_t + \varepsilon_{t-1} + \cdots + \varepsilon_1$

$y_t^2 = (y_0 + \varepsilon_t + \varepsilon_{t-1} + \cdots + \varepsilon_1)(y_0 + \varepsilon_t + \varepsilon_{t-1} + \cdots + \varepsilon_1)$

$= y_0^2 + 2y_0(\varepsilon_t + \cdots + \varepsilon_1)$

$+ \boxed{\varepsilon_t^2 + \varepsilon_{t-1}^2 + \cdots + \varepsilon_1^2} +$ CROSS-PRODUCTS IN ε_{t-i}

AND $E(y_t^2) = \sigma^2_\varepsilon t$ + OTHER TERMS $\to \infty$ AS $t \to \infty$.

2f) $y_t = \alpha_0 + \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2}$;

$t=1,2,\ldots,T$; $\varepsilon_t \sim IID(0, \sigma^2_\varepsilon)$; y_0 INITIAL.

SO $y_t \sim MA(2)$.

ALL MA(q) ARE STATIONARY (#138), SINCE ONLY THE INVERSE (EXISTENCE OF...) OF A(L) MATTERS; HERE,

$A(L) y_t = \alpha_0 + B(L)\varepsilon_t$, WITH

$A(L) = 1$

$B(L) = 1 + \beta_1 L + \beta_2 L^2$

SO $\underline{A(L)^{-1} = 1}$ ... :)