IENAC17/ Forecasting Final Examination

Date: Monday 27 May 2019.

Time allowed: 2 hours (08:00-10:00).

Answer all questions briefly.

Show all computations (including relevant critical values).

You are allowed all lecture handouts and notes, but no textbooks.

An English–French dictionary is allowed, as is a scientific calculator.

Question 1 is for 70 marks. Question 2 is for 50 marks.

You are given a cross-sectional dataset with 50 observations. It contains information on the state-by-state outcomes of the 1992 U.S. Presidential Election. (There were three major candidates: the incumbent Republican President George H. W. Bush, the Democratic Arkansas Governor Bill Clinton, and the independent Texas businessman Ross Perot. Bill Clinton won a *plurality* (43%) of the nationwide vote, i.e. the largest number of votes by any candidate, without gaining an absolute majority of 50%. He also won a large majority of electoral college votes, and was elected 42nd President of the United States.) The variables are DVOTE (the percentage of votes cast for the Democratic candidate in the 1992 election), DEM (= 1 if the Democratic candidate received a plurality of the state vote), UN (the state unemployment rate in 1992), FEDFUNDS (per capita federal spending in 1992, i.e. government expenditure by state), DEFENSE (defense contract awards per 1000 population in 1992, i.e. number of U.S. Department of Defense contracts awarded to each state), CRIME (total crime rate per 100,000 population in 1992, by state), NE (= 1 for northeastern states), S (= 1 for southern states), W (= 1 for western states). Note that the location dummies are not exhaustive. **This is used in Question 1.** You are also given a time-series dataset with 170 observations, on the quarterly U.S. nominal interest rate (percentage), denoted Y, from 1960Q1 to 2002Q2. **This is used in Question 2.**

Question 1

- This question uses the *1992 U.S. Presidential Election* data (**refer to Figures 1–10**). We will use this data to construct models to explain whether the Democratic candidate received a plurality of votes in a state.

(a) Perform a brief preliminary analysis of the variables, and explain your findings. Refer to the descriptive statistics, bivariate correlations, and histograms.

(10 marks)

(b) Discuss the output from the probit model (EQ0a) for DEM.

(10 marks)

(c) For EQ0a, compute the estimated marginal effects evaluated at the mean. You do **not** need to interpret their economic significance.

(15 marks)

(d) For EQ0a, interpret the response curves of estimated probability against UN, for (S= 0 and W= 0), (S= 0 and W= 1), and (S= 1 and W= 0).

(5 marks)

(e) For EQ0a, discuss the table of hits-and-misses. What would be the overall percentage of correct predictions for the "naïve" model?

(5 marks)

(f) In the logit model (EQ0b) for DEM, why are the estimated coefficients larger than those in the probit model EQ0a?

(5 marks)

(g) For EQ0b, compute the estimated odds-ratio at the mean.

(10 marks)

(h) Why is the log-likelihood always negative in a discrete choice model?

(5 marks)

(i) For EQ0c, test rigorously for first-order autocorrelation using the asymptotic distribution of the Durbin-Watson statistic. Does this agree with Breusch-Godfrey?

(5 marks)

questions continue overleaf

Question 2

- This question uses the *U.S. interest rate* data (**refer to Figures 11–34**). We use this data to construct a medium-term dynamic forecast of the nominal interest rate.

(a) Discuss the validity and quality of the ARMA(1,2) model for Y_t.

(10 marks)

(b) Discuss the validity of the MA(2) model for ΔY_t, and the quality of its forecast.

(15 marks)

(c) Write down the theoretical ACFs for the MA(1) and MA(2) in EQ04 and EQ05. Hint: You are not required to give a formal proof here.

(5 marks)

(d) Using your answer to question (2c), substitute in the estimated coefficients from EQ04 and EQ05 and simplify, giving your answers to 1 decimal place.

(5 marks)

(e) Show that the following model is nonstationary:

$y_t = y_{t-1} + \varepsilon_t; \quad t = 1, 2, \ldots, T; \quad \varepsilon_t \sim \text{i.i.d.}(0, \sigma_\varepsilon^2); \quad \text{initial value } y_0.$

(10 marks)

(f) Explain why the following model is stationary:

$y_t = \alpha_0 + \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2}; \quad t = 1, 2, \ldots, T; \quad \varepsilon_t \sim \text{i.i.d.}(0, \sigma_\varepsilon^2); \quad \text{initial value } y_0.$

(5 marks)

	DVOTE	DEM	UN	FEDFUNDS	DEFENSE	CRIME	NE	S	W
Mean	42.16600	0.660000	6.822000	5181.140	0.434920	5221.800	0.180000	0.340000	0.160000
Median	42.60000	1.000000	6.900000	4446.000	0.317000	5116.500	0.000000	0.000000	0.000000
Maximum	84.60000	1.000000	11.30000	33974.00	2.625000	11407.00	1.000000	1.000000	1.000000
Minimum	24.70000	0.000000	3.000000	3529.000	0.046000	2610.000	0.000000	0.000000	0.000000
Std. Dev.	8.522853	0.478518	1.598633	4228.791	0.422128	1509.291	0.388088	0.478518	0.370328
Skewness	2.172037	-0.675521	-0.125485	6.497942	3.012479	1.262779	1.665853	0.675521	1.854852
Kurtosis	13.69697	1.456328	3.475150	44.76117	15.56413	7.107213	3.775068	1.456328	4.440476
Jarque-Bera	277.7002	8.767157	0.601570	3985.184	404.4950	48.43259	24.37709	8.767157	32.99349
Probability	0.000000	0.012481	0.740237	0.000000	0.000000	0.000000	0.000005	0.012481	0.000000
Sum	2108.300	33.00000	341.1000	259057.0	21.74600	261090.0	9.000000	17.00000	8.000000
Sum Sq. Dev.	3559.312	11.22000	125.2258	8.76E+08	8.731408	1.12E+08	7.380000	11.22000	6.720000
Observations	50	50	50	50	50	50	50	50	50

Figure 1: Descriptive statistics.

Correlation	DVOTE	DEM	UN	FEDFUN...	DEFENSE	CRIME	NE	S	W
DVOTE	1.000000								
DEM	0.564065	1.000000							
UN	0.373022	0.295434	1.000000						
FEDFUNDS	0.710951	0.085568	0.162018	1.000000					
DEFENSE	0.570452	0.068060	0.235691	0.810283	1.000000				
CRIME	0.460535	-0.003063	0.212977	0.591465	0.547270	1.000000			
NE	0.116034	0.336277	0.299407	-0.057815	0.119931	-0.279856	1.000000		
S	0.292626	-0.197861	0.171434	0.200139	0.161284	0.251050	-0.336277	1.000000	
W	-0.280157	-0.032246	-0.078459	-0.046551	-0.093259	0.156114	-0.204479	-0.313248	1.000000

Figure 2: Bivariate correlations.

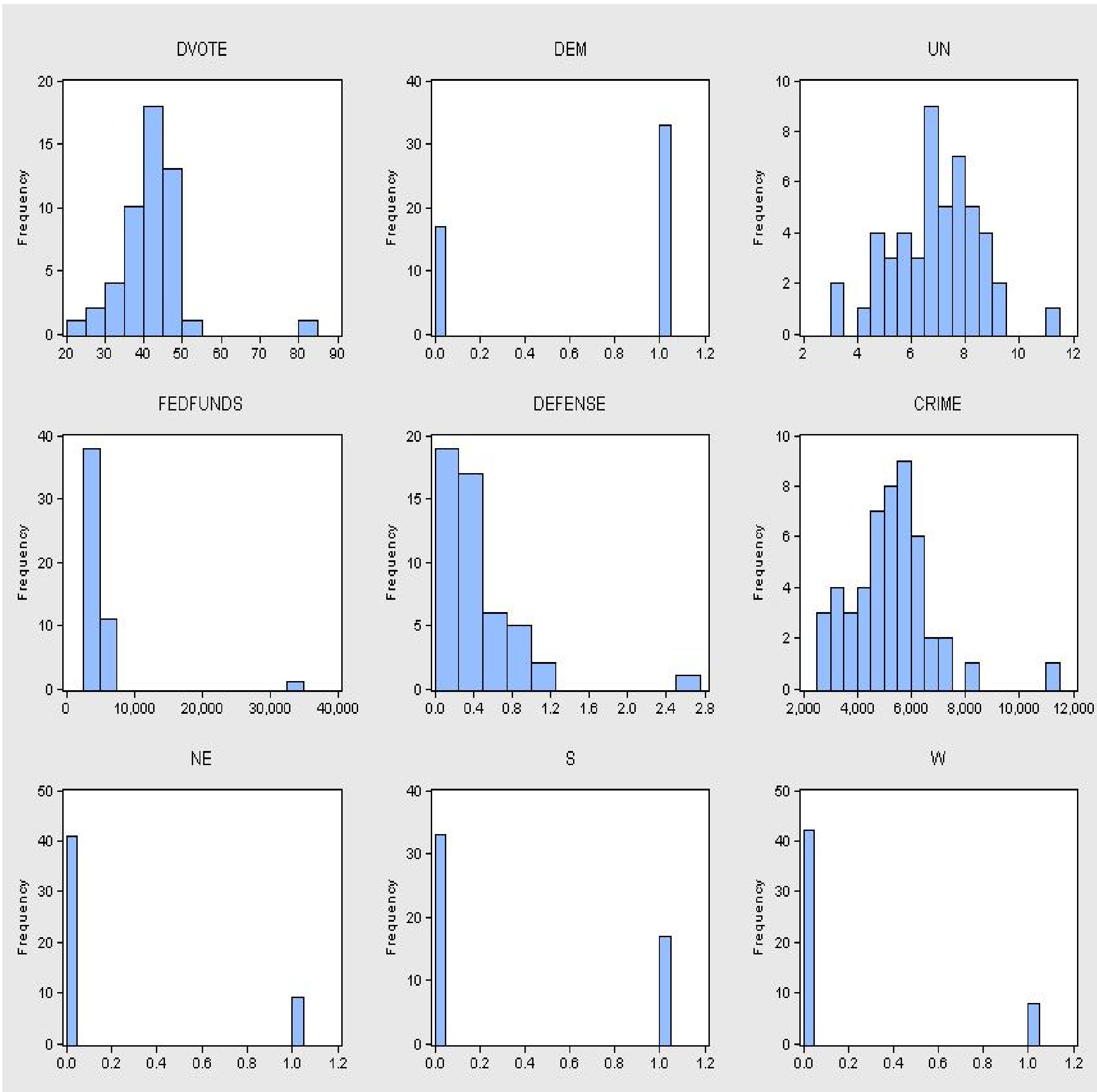


Figure 3: Histograms.

Dependent Variable: DEM
Method: ML - Binary Probit (Quadratic hill climbing)
Date: 04/25/19 Time: 15:07
Sample: 1 50
Included observations: 50
Convergence achieved after 5 iterations
Covariance matrix computed using second derivatives

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-1.340666	1.075362	-1.246711	0.2125
CRIME	-5.11E-05	0.000183	-0.279716	0.7797
DEFENSE	-0.467755	0.864660	-0.540970	0.5885
FEDFUNDS	8.26E-05	0.000104	0.798126	0.4248
UN	0.329023	0.144984	2.269382	0.0232
S	-0.873189	0.470434	-1.856135	0.0634
W	-0.418786	0.585116	-0.715733	0.4742

McFadden R-squared	0.146585	Mean dependent var	0.660000
S.D. dependent var	0.478518	S.E. of regression	0.460428
Akaike info criterion	1.374139	Sum squared resid	9.115751
Schwarz criterion	1.641822	Log likelihood	-27.35348
Hannan-Quinn criter.	1.476075	Restr. log likelihood	-32.05177
LR statistic	9.396587	Avg. log likelihood	-0.547070
Prob(LR statistic)	0.152472		

Obs with Dep=0	17	Total obs	50
Obs with Dep=1	33		

Figure 4: EQ0a: Probit model.

	C	CRIME	DEFENSE	FEDFUNDS	UN	S	W
C	1.156403	-9.39E-05	0.254631	-1.95E-05	-0.103730	0.082223	0.062564
CRIME	-9.39E-05	3.34E-08	-1.99E-05	-4.20E-09	-5.34E-06	-2.68E-05	-3.65E-05
DEFENSE	0.254631	-1.99E-05	0.747638	-5.86E-05	-0.029943	0.025930	0.068923
FEDFUNDS	-1.95E-05	-4.20E-09	-5.86E-05	1.07E-08	2.00E-06	2.23E-07	7.25E-07
UN	-0.103730	-5.34E-06	-0.029943	2.00E-06	0.021020	-0.008980	-0.001773
S	0.082223	-2.68E-05	0.025930	2.23E-07	-0.008980	0.221308	0.120458
W	0.062564	-3.65E-05	0.068923	7.25E-07	-0.001773	0.120458	0.342360

Figure 5: EQ0a: Coefficient covariance matrix from probit model.

Dependent Variable: DEM
Method: ML - Binary Logit (Quadratic hill climbing)
Date: 04/25/19 Time: 15:10
Sample: 1 50
Included observations: 50
Convergence achieved after 5 iterations
Covariance matrix computed using second derivatives

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-2.442380	1.885387	-1.295426	0.1952
CRIME	-8.02E-05	0.000309	-0.259889	0.7949
DEFENSE	-0.818580	1.438882	-0.568900	0.5694
FEDFUNDS	0.000148	0.000196	0.756648	0.4493
UN	0.580741	0.260394	2.230235	0.0257
S	-1.538007	0.819627	-1.876472	0.0606
W	-0.776971	0.994041	-0.781629	0.4344

McFadden R-squared	0.150361	Mean dependent var	0.660000
S.D. dependent var	0.478518	S.E. of regression	0.459719
Akaike info criterion	1.369298	Sum squared resid	9.087688
Schwarz criterion	1.636981	Log likelihood	-27.23244
Hannan-Quinn criter.	1.471233	Restr. log likelihood	-32.05177
LR statistic	9.638664	Avg. log likelihood	-0.544649
Prob(LR statistic)	0.140717		

Obs with Dep=0	17	Total obs	50
Obs with Dep=1	33		

Figure 6: EQ0b: Logit model.

Dependent Variable: DVOTE
Method: Least Squares
Date: 04/25/19 Time: 15:12
Sample: 1 50
Included observations: 50

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	24.88061	4.196474	5.928934	0.0000
CRIME	0.000546	0.000698	0.782597	0.4382
DEFENSE	-3.040407	3.268673	-0.930166	0.3575
FEDFUNDS	0.001448	0.000333	4.353515	0.0001
UN	1.317571	0.511752	2.574629	0.0136
S	0.516183	1.828294	0.282330	0.7790
W	-5.693668	2.356125	-2.416539	0.0200

R-squared	0.639370	Mean dependent var	42.16600
Adjusted R-squared	0.589050	S.D. dependent var	8.522853
S.E. of regression	5.463608	Akaike info criterion	6.363273
Sum squared resid	1283.594	Schwarz criterion	6.630956
Log likelihood	-152.0818	Hannan-Quinn criter.	6.465208
F-statistic	12.70598	Durbin-Watson stat	1.587490
Prob(F-statistic)	0.000000		

Figure 7: EQ0c: Linear model for DVOTE.

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	2.623201	Prob. F(2,41)	0.0847
Obs*R-squared	5.672227	Prob. Chi-Square(2)	0.0587

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 04/25/19 Time: 17:58
Sample: 1 50
Included observations: 50
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.720123	4.013664	-0.179418	0.8585
CRIME	0.000109	0.000662	0.164518	0.8701
DEFENSE	0.104377	3.103269	0.033635	0.9733
FEDFUNDS	2.68E-05	0.000315	0.085038	0.9326
UN	0.004252	0.484245	0.008782	0.9930
S	-0.227405	1.744465	-0.130358	0.8969
W	-0.017812	2.244846	-0.007935	0.9937
RESID(-1)	0.106647	0.150560	0.708337	0.4827
RESID(-2)	-0.334117	0.147816	-2.260359	0.0292

R-squared	0.113445	Mean dependent var	-6.04E-16
Adjusted R-squared	-0.010261	S.D. dependent var	5.118182
S.E. of regression	5.144374	Akaike info criterion	6.322862
Sum squared resid	1137.977	Schwarz criterion	6.667026
Log likelihood	-149.0715	Hannan-Quinn criter.	6.453921
F-statistic	0.687790	Durbin-Watson stat	1.683441
Prob(F-statistic)	0.699852		

Figure 8: EQ0c: Breusch-Godfrey test results.

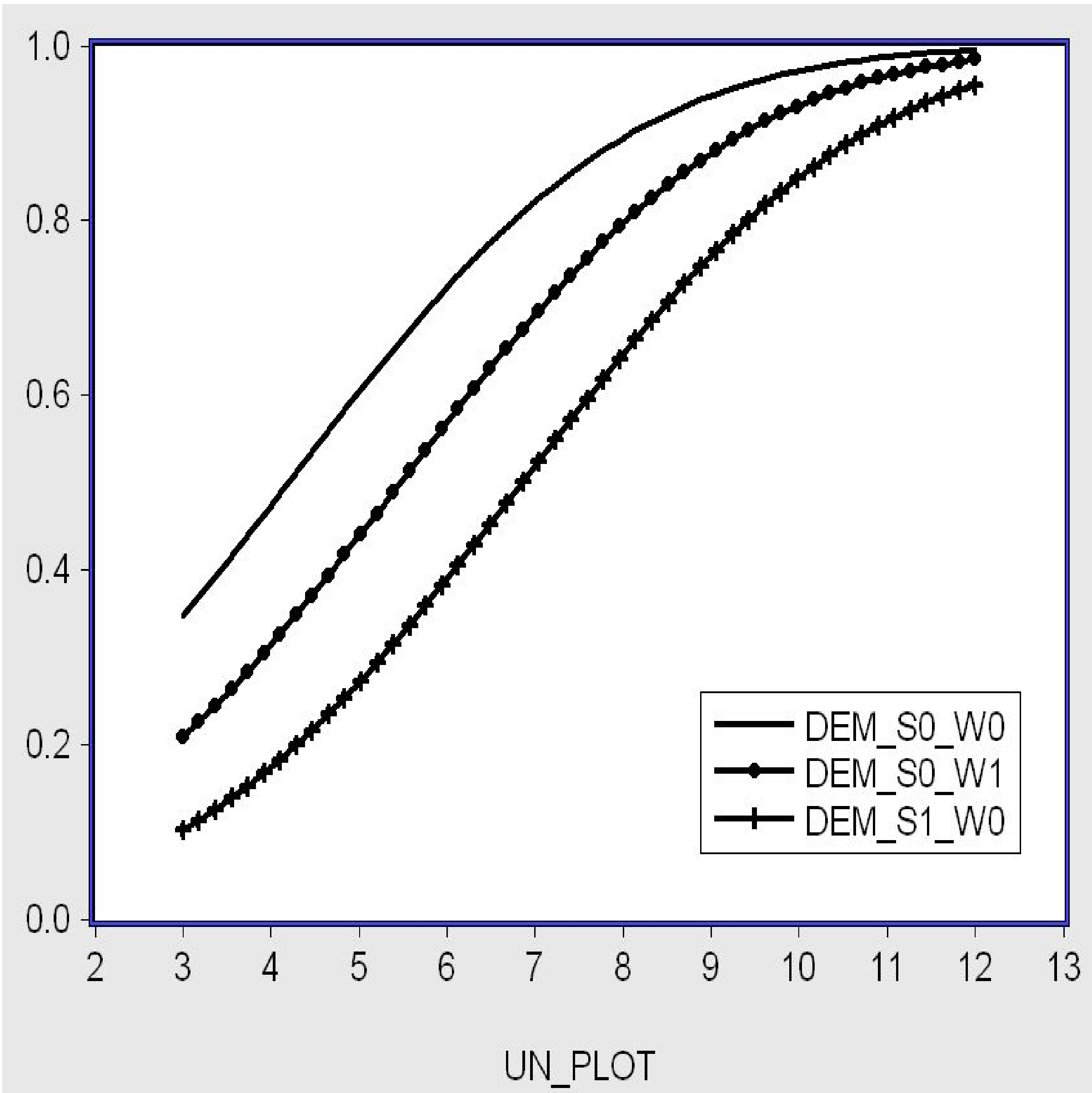


Figure 9: EQ0a: Probability response curves from probit: DEM_S0_W0 corresponds to (S=0 and W=0); DEM_S0_W1 corresponds to (S=0 and W=1); DEM_S1_W0 corresponds to (S=1 and W=0); UN_PLOT corresponds to the unemployment rate.

Success cutoff: C = 0.5

	Estimated Equation		
	Dep=0	Dep=1	Total
P(Dep=1)<=C	9	5	14
P(Dep=1)>C	8	28	36
Total	17	33	50
Correct	9	28	37
% Correct	52.94	84.85	74.00
% Incorrect	47.06	15.15	26.00

Figure 10: EQ0a: Table of hits-and-misses from probit model.

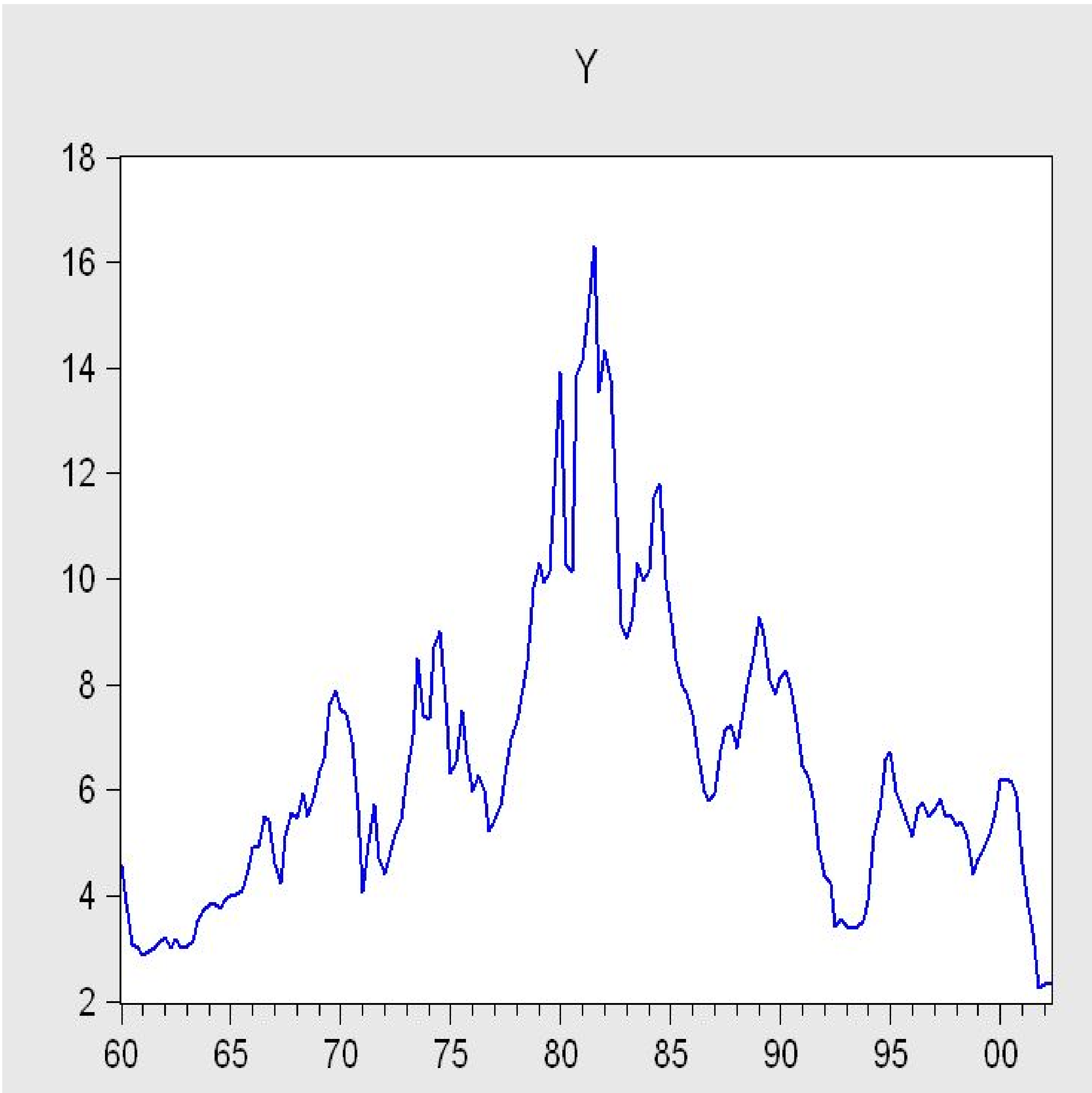


Figure 11: Interest rate Y_t.

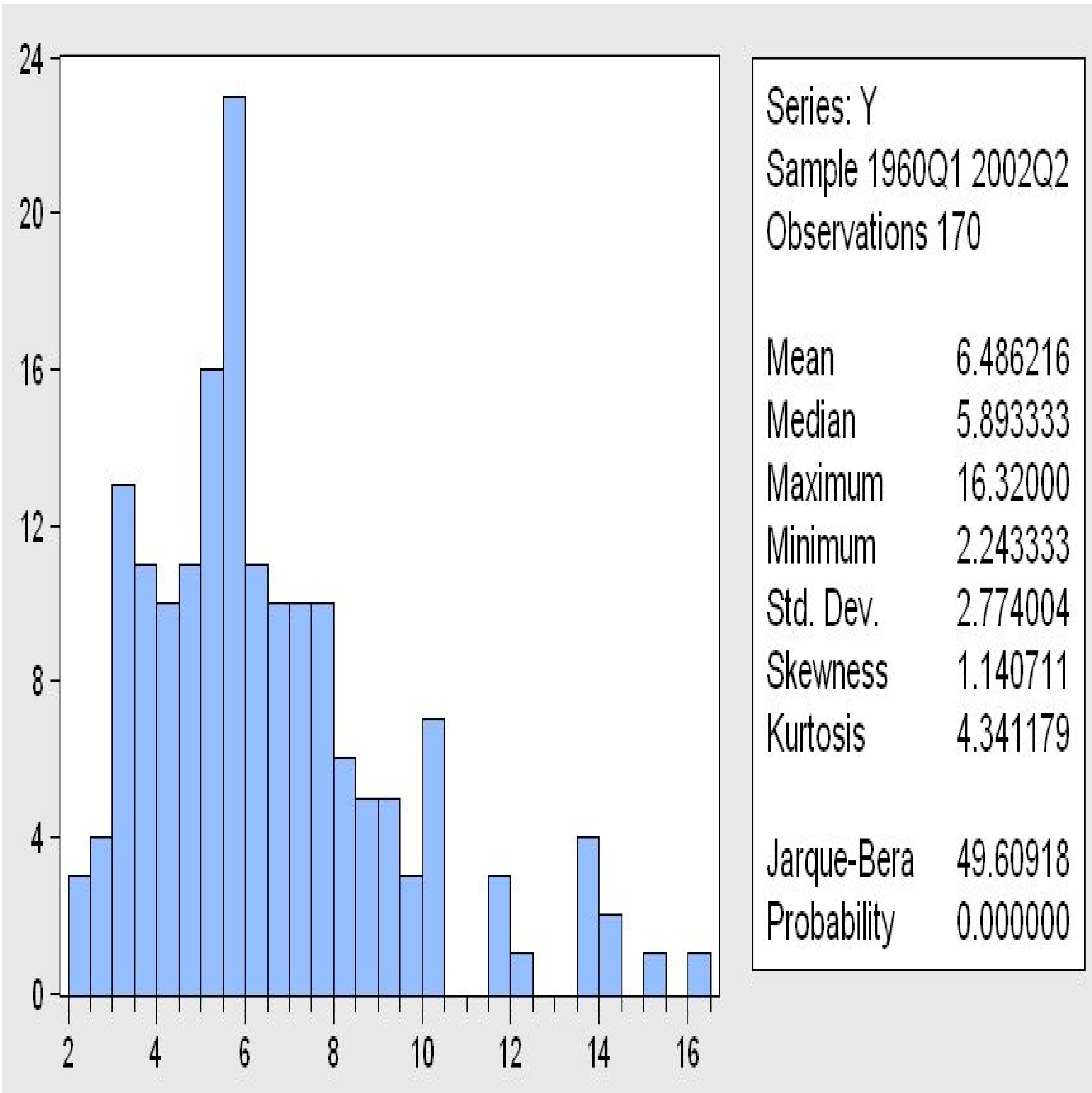
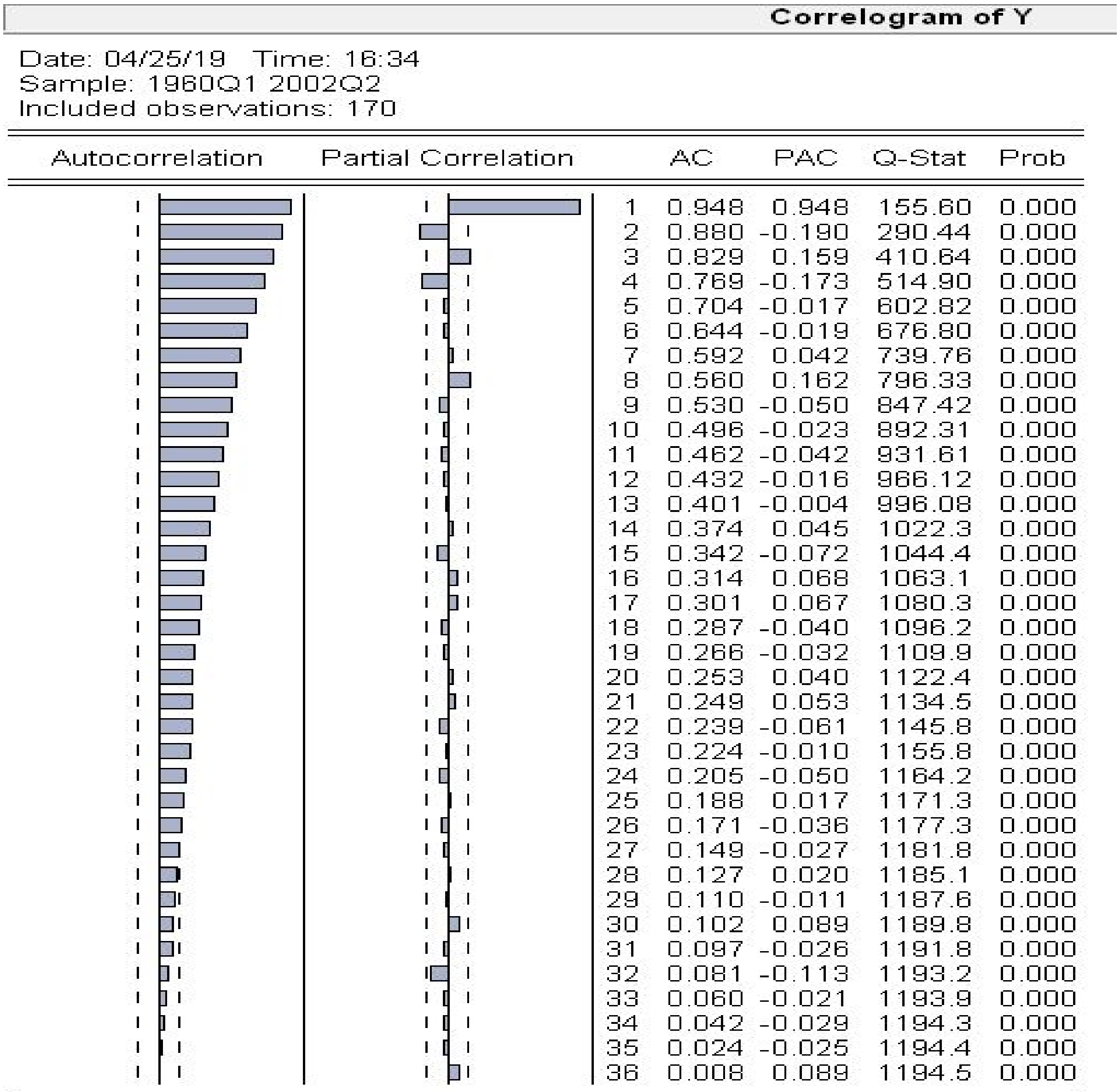


Figure 12: Descriptive statistics.



Correlogram of Y

Date: 04/25/19 Time: 16:34
Sample: 1960Q1 2002Q2
Included observations: 170

Lag	AC	PAC	Q-Stat	Prob
1	0.948	0.948	155.60	0.000
2	0.880	-0.190	290.44	0.000
3	0.829	0.159	410.64	0.000
4	0.769	-0.173	514.90	0.000
5	0.704	-0.017	602.82	0.000
6	0.644	-0.019	676.80	0.000
7	0.592	0.042	739.76	0.000
8	0.560	0.162	796.33	0.000
9	0.530	-0.050	847.42	0.000
10	0.496	-0.023	892.31	0.000
11	0.462	-0.042	931.61	0.000
12	0.432	-0.016	966.12	0.000
13	0.401	-0.004	996.08	0.000
14	0.374	0.045	1022.3	0.000
15	0.342	-0.072	1044.4	0.000
16	0.314	0.068	1063.1	0.000
17	0.301	0.067	1080.3	0.000
18	0.287	-0.040	1096.2	0.000
19	0.266	-0.032	1109.9	0.000
20	0.253	0.040	1122.4	0.000
21	0.249	0.053	1134.5	0.000
22	0.239	-0.061	1145.8	0.000
23	0.224	-0.010	1155.8	0.000
24	0.205	-0.050	1164.2	0.000
25	0.188	0.017	1171.3	0.000
26	0.171	-0.036	1177.3	0.000
27	0.149	-0.027	1181.8	0.000
28	0.127	0.020	1185.1	0.000
29	0.110	-0.011	1187.6	0.000
30	0.102	0.089	1189.8	0.000
31	0.097	-0.026	1191.8	0.000
32	0.081	-0.113	1193.2	0.000
33	0.060	-0.021	1193.9	0.000
34	0.042	-0.029	1194.3	0.000
35	0.024	-0.025	1194.4	0.000
36	0.008	0.089	1194.5	0.000

Figure 13: Correlogram.

Dependent Variable: Y
Method: Least Squares
Date: 04/25/19 Time: 16:36
Sample (adjusted): 1960Q2 2002Q2
Included observations: 169 after adjustments
Convergence achieved after 4 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	6.172535	1.626258	3.795544	0.0002
AR(1)	0.961042	0.022821	42.11165	0.0000

R-squared	0.913935	Mean dependent var	6.497554
Adjusted R-squared	0.913420	S.D. dependent var	2.778294
S.E. of regression	0.817501	Akaike info criterion	2.446634
Sum squared resid	111.6073	Schwarz criterion	2.483674
Log likelihood	-204.7405	Hannan-Quinn criter.	2.461665
F-statistic	1773.391	Durbin-Watson stat	1.609609
Prob(F-statistic)	0.000000		

Inverted AR Roots	.96

Figure 14: EQ01: AR(1) model.

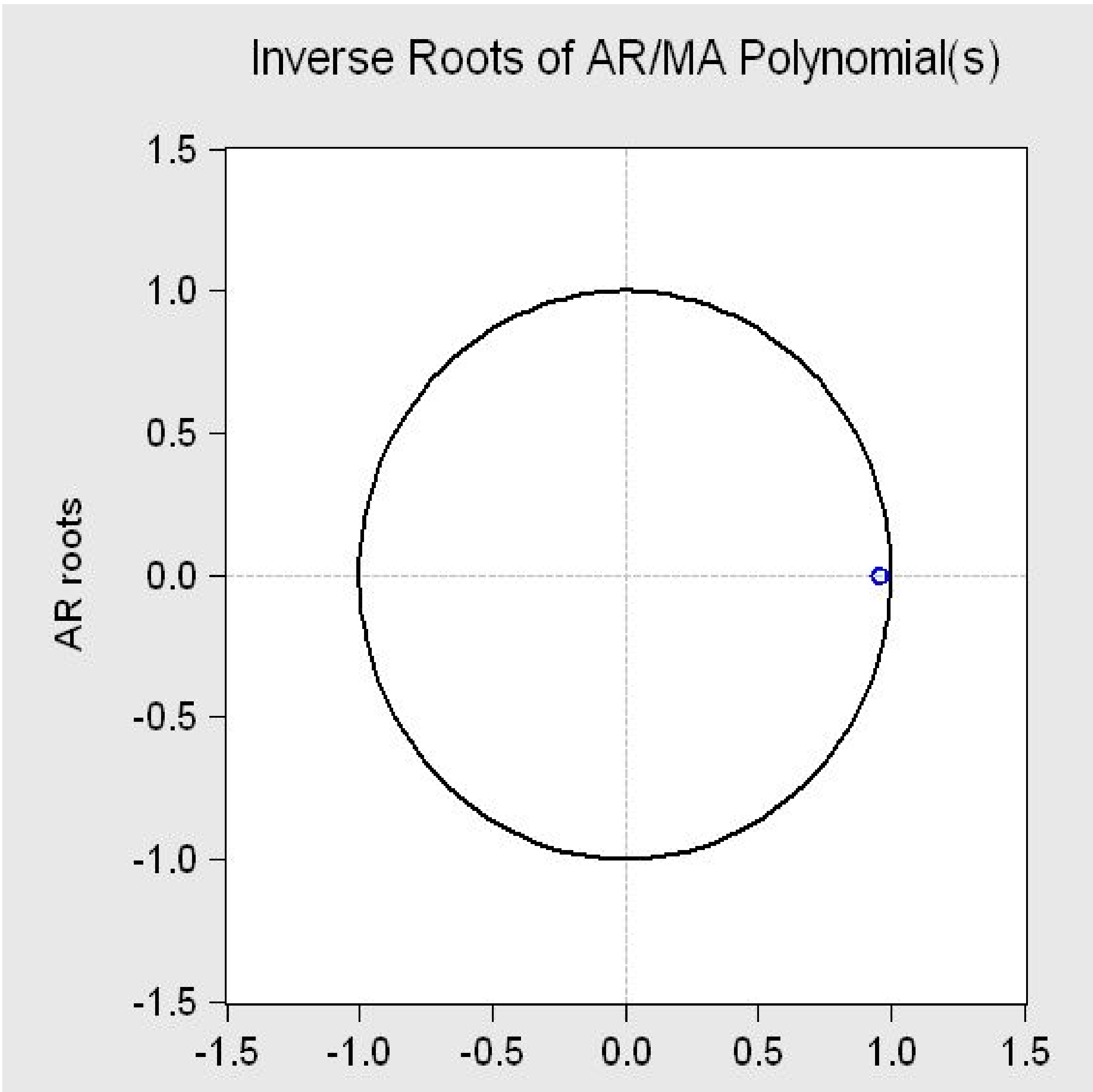


Figure 15: EQ01: AR(1) estimated root λ.

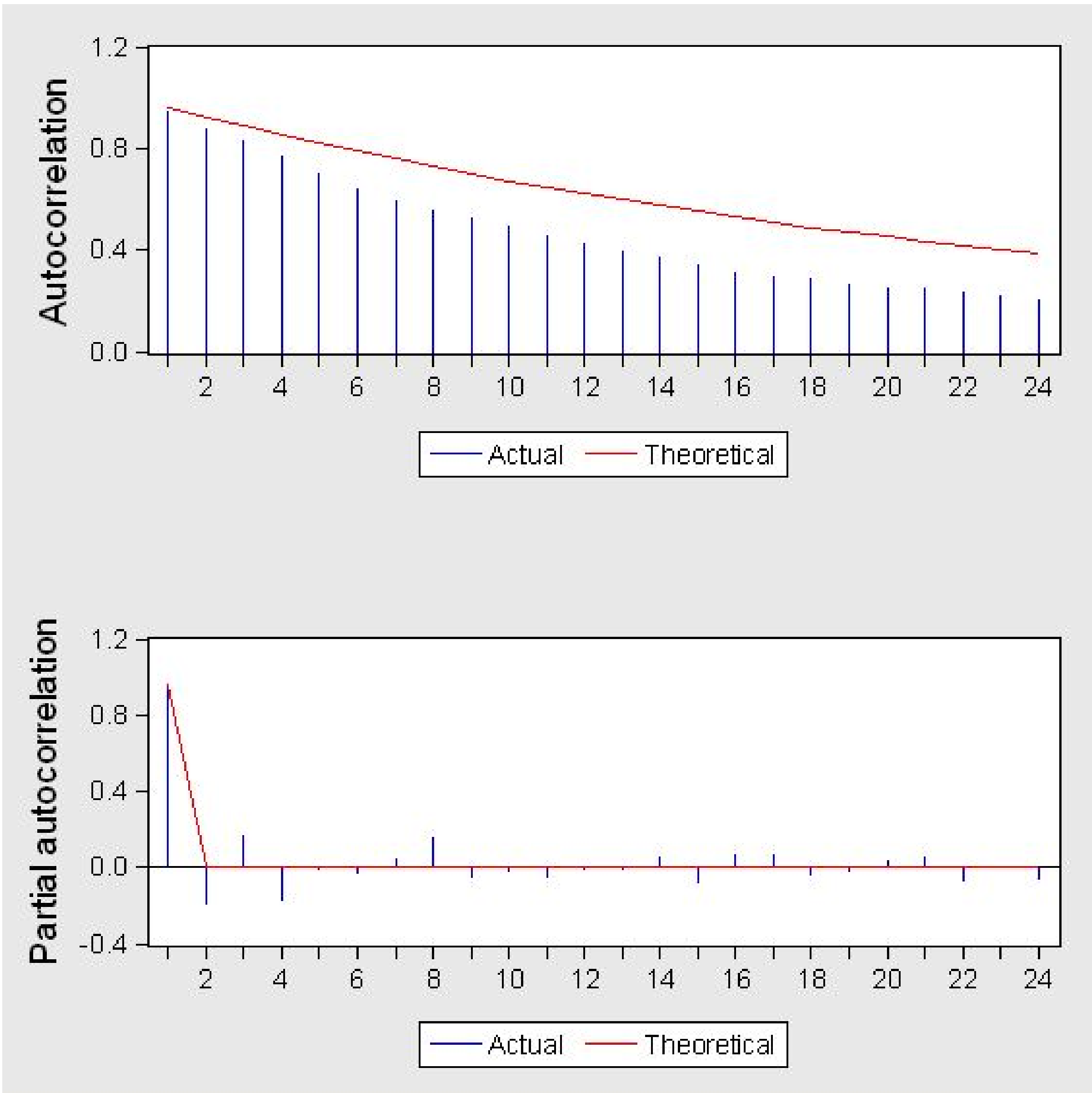
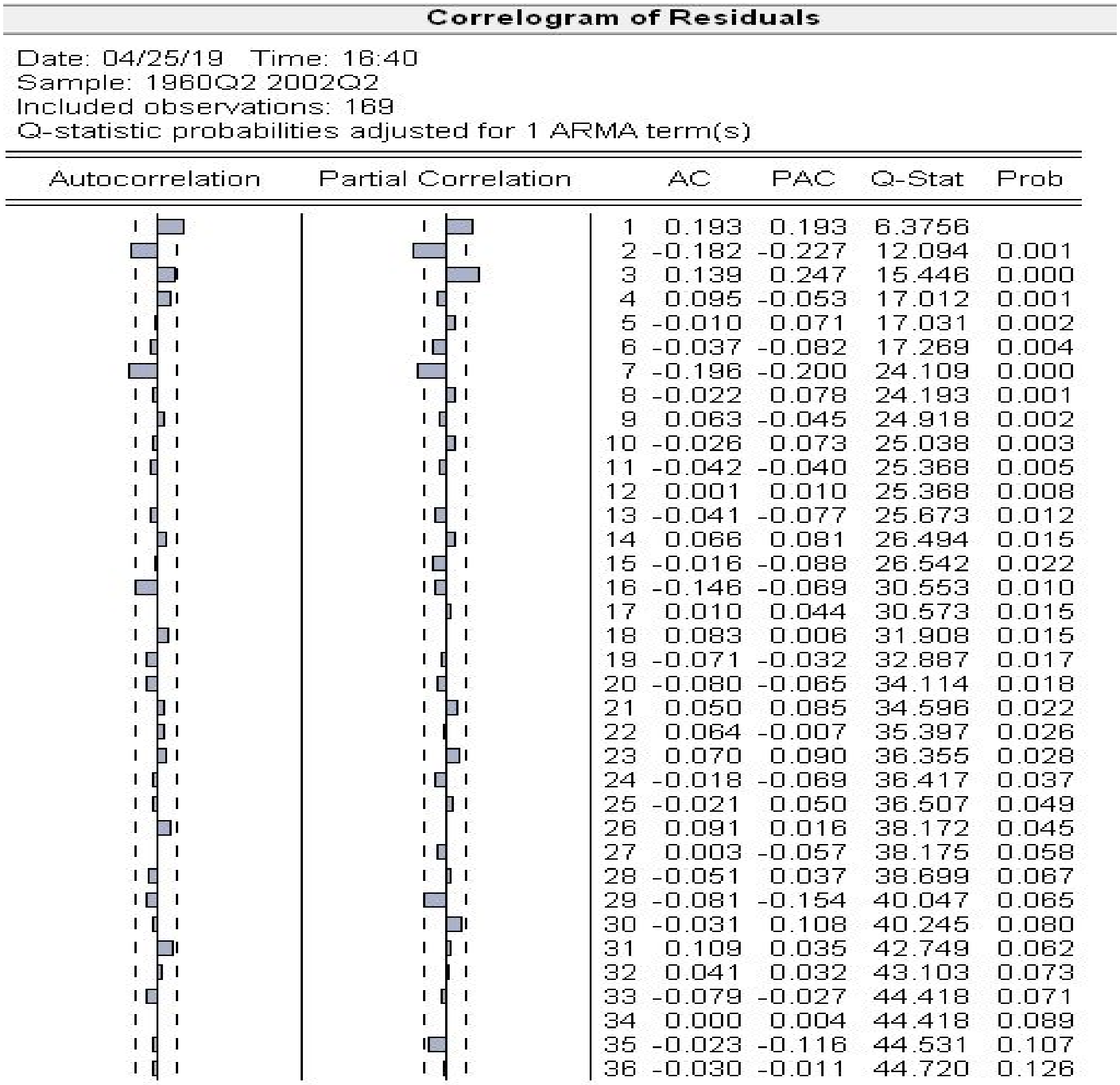


Figure 16: EQ01: AR(1) estimated correlograms.



Correlogram of Residuals

Date: 04/25/19 Time: 16:40
Sample: 1960Q2 2002Q2
Included observations: 169
Q-statistic probabilities adjusted for 1 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.193	0.193	6.3756	
		2	-0.182	-0.227	12.094	0.001
		3	0.139	0.247	15.446	0.000
		4	0.095	-0.053	17.012	0.001
		5	-0.010	0.071	17.031	0.002
		6	-0.037	-0.082	17.269	0.004
		7	-0.196	-0.200	24.109	0.000
		8	-0.022	0.078	24.193	0.001
		9	0.063	-0.045	24.918	0.002
		10	-0.026	0.073	25.038	0.003
		11	-0.042	-0.040	25.368	0.005
		12	0.001	0.010	25.368	0.008
		13	-0.041	-0.077	25.673	0.012
		14	0.066	0.081	26.494	0.015
		15	-0.016	-0.088	26.542	0.022
		16	-0.146	-0.069	30.553	0.010
		17	0.010	0.044	30.573	0.015
		18	0.083	0.006	31.908	0.015
		19	-0.071	-0.032	32.887	0.017
		20	-0.080	-0.065	34.114	0.018
		21	0.050	0.085	34.596	0.022
		22	0.064	-0.007	35.397	0.026
		23	0.070	0.090	36.355	0.028
		24	-0.018	-0.069	36.417	0.037
		25	-0.021	0.050	36.507	0.049
		26	0.091	0.016	38.172	0.045
		27	0.003	-0.057	38.175	0.058
		28	-0.051	0.037	38.699	0.067
		29	-0.081	-0.154	40.047	0.065
		30	-0.031	0.108	40.245	0.080
		31	0.109	0.035	42.749	0.062
		32	0.041	0.032	43.103	0.073
		33	-0.079	-0.027	44.418	0.071
		34	0.000	0.004	44.418	0.089
		35	-0.023	-0.116	44.531	0.107
		36	-0.030	-0.011	44.720	0.126

Figure 17: EQ01: AR(1) residual correlograms.

Dependent Variable: Y
Method: Least Squares
Date: 04/25/19 Time: 16:43
Sample (adjusted): 1960Q2 2002Q2
Included observations: 169 after adjustments
Convergence achieved after 13 iterations
MA Backcast: 1959Q4 1960Q1

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	6.222825	1.508775	4.124421	0.0001
AR(1)	0.955346	0.027124	35.22087	0.0000
MA(1)	0.320221	0.082711	3.871557	0.0002
MA(2)	-0.190454	0.082529	-2.307712	0.0223

R-squared	0.924632	Mean dependent var	6.497554
Adjusted R-squared	0.923262	S.D. dependent var	2.778294
S.E. of regression	0.769633	Akaike info criterion	2.337578
Sum squared resid	97.73530	Schwarz criterion	2.411659
Log likelihood	-193.5254	Hannan-Quinn criter.	2.367642
F-statistic	674.7545	Durbin-Watson stat	2.006687
Prob(F-statistic)	0.000000		

Inverted AR Roots	.96	
Inverted MA Roots	.30	-.62

Figure 18: EQ02: ARMA(1,2) model.

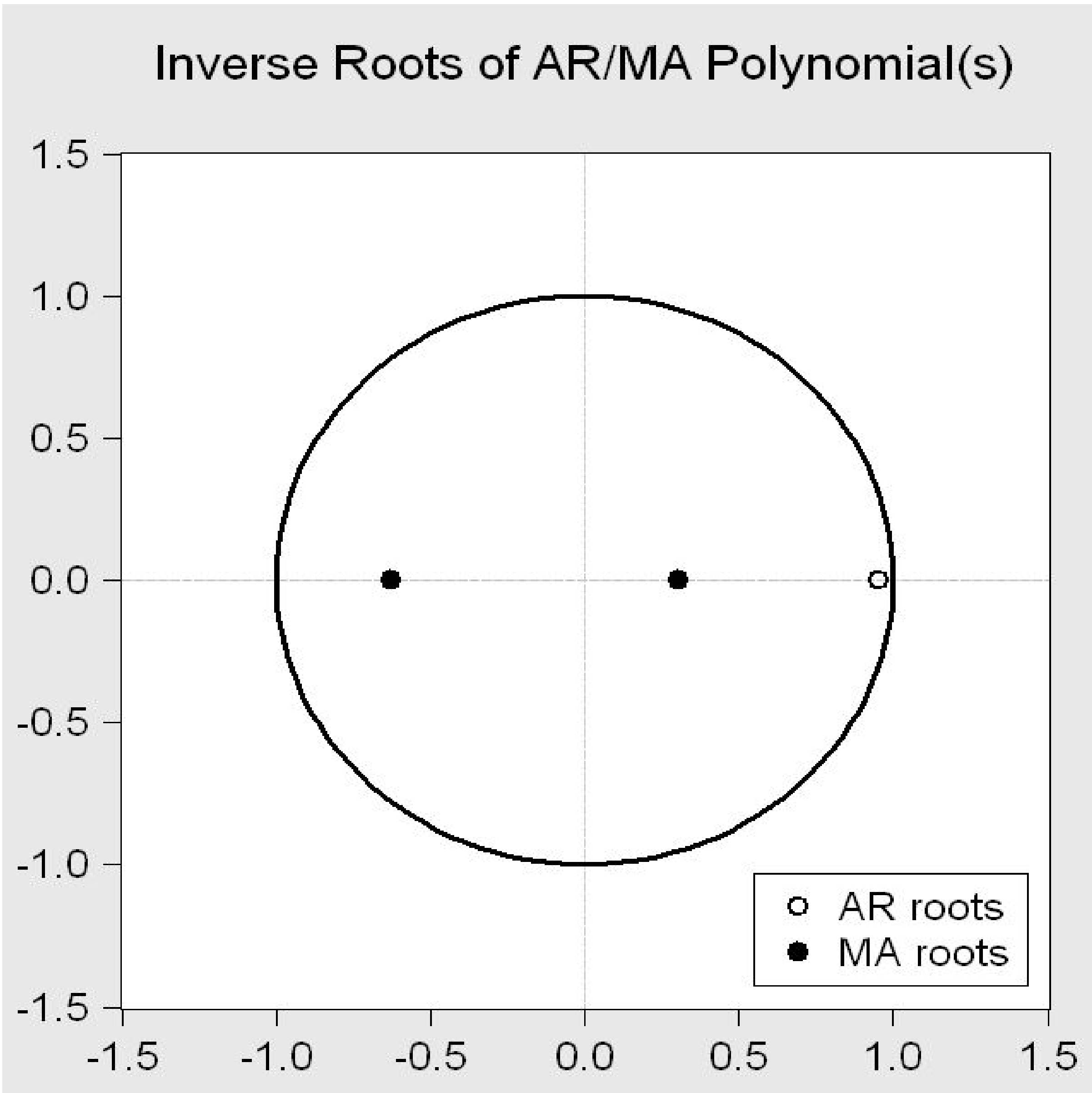


Figure 19: EQ02: ARMA(1,2) estimated roots λ.

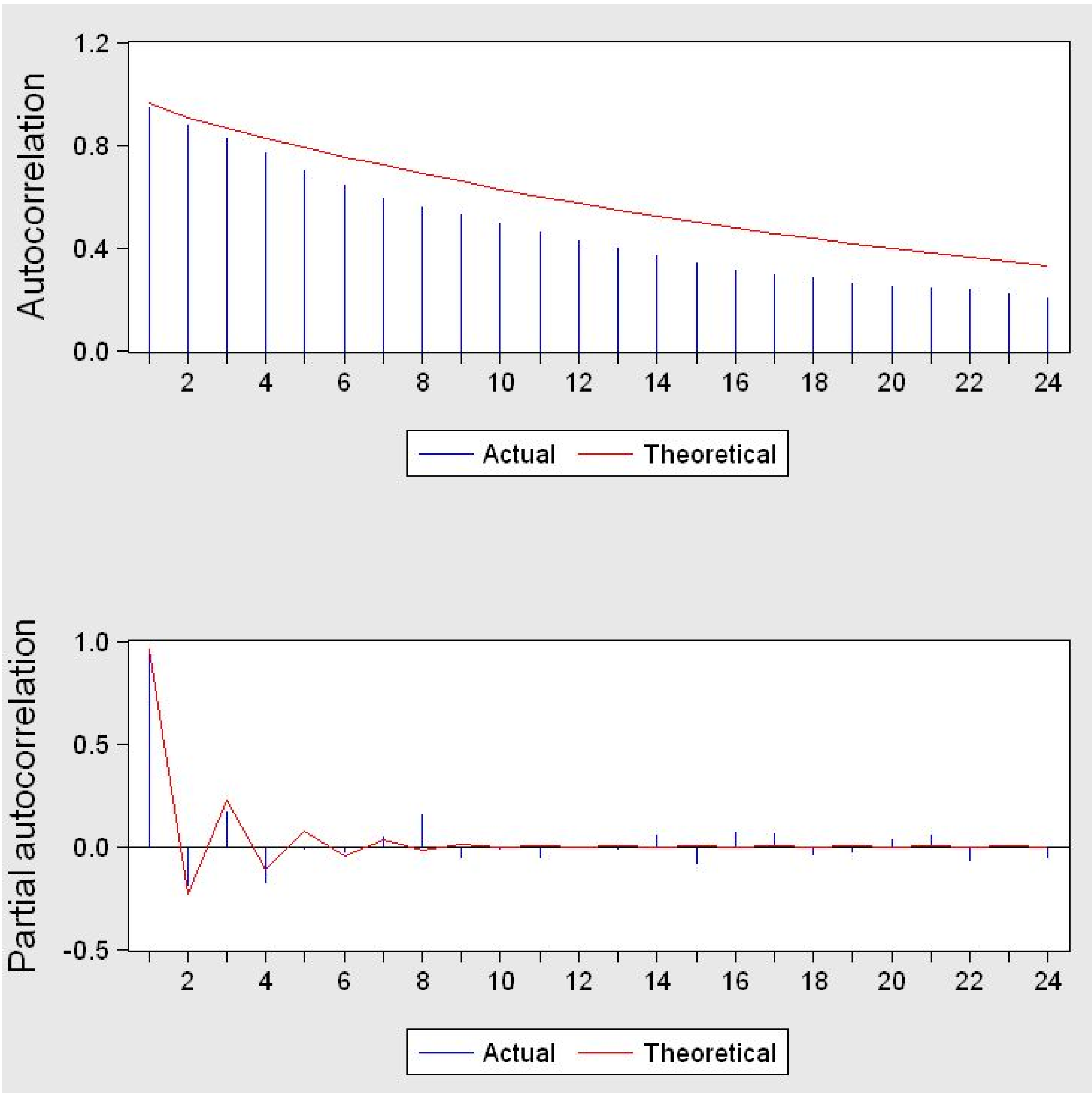


Figure 20: EQ02: ARMA(1,2) estimated correlograms.

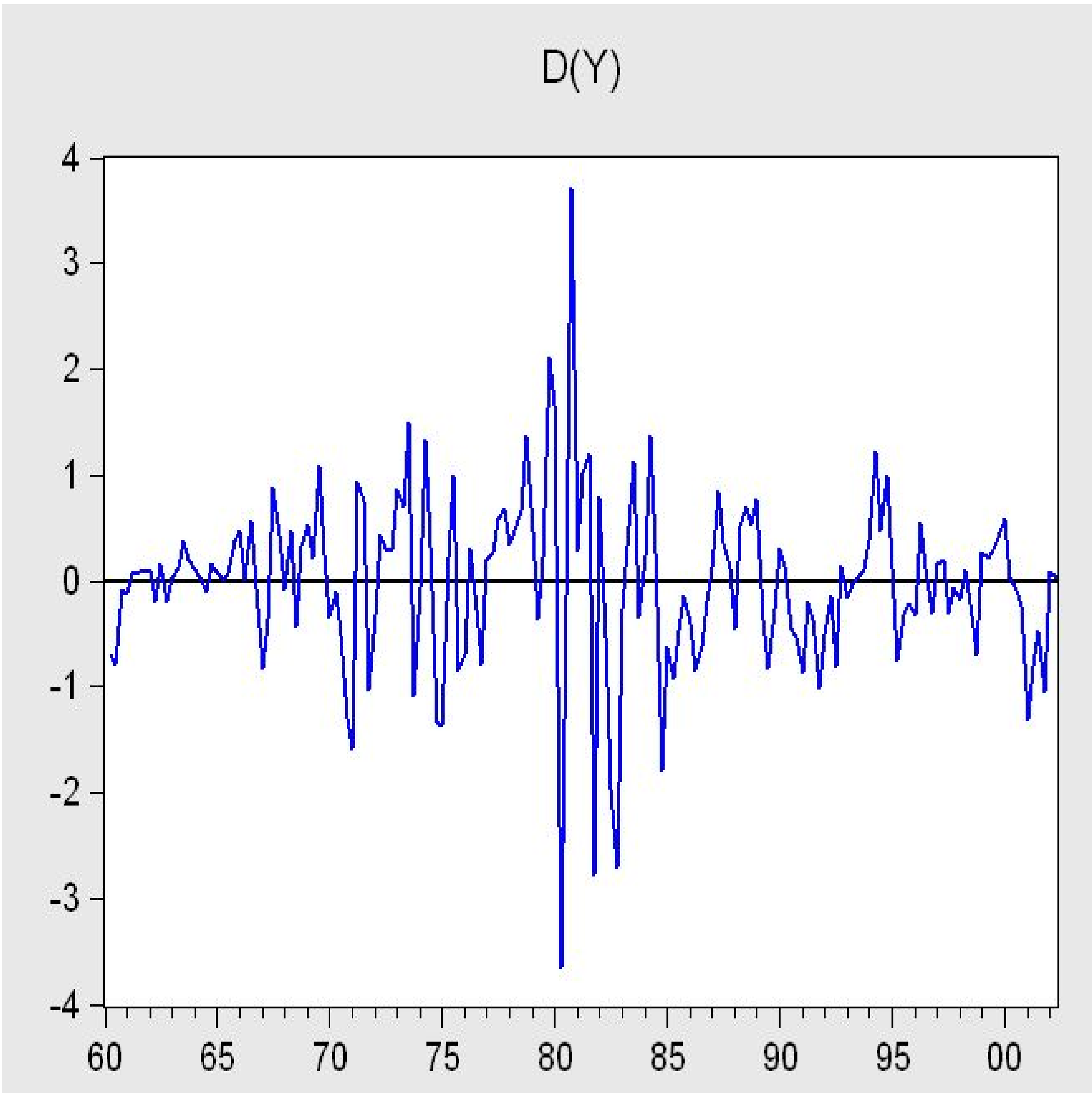


Figure 21: Plot of ΔY_t.

Dependent Variable: D(Y)
Method: Least Squares
Date: 04/25/19 Time: 16:50
Sample (adjusted): 1960Q2 2002Q2
Included observations: 169 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.013176	0.063242	-0.208336	0.8352

R-squared	0.000000	Mean dependent var	-0.013176
Adjusted R-squared	0.000000	S.D. dependent var	0.822145
S.E. of regression	0.822145	Akaike info criterion	2.452099
Sum squared resid	113.5549	Schwarz criterion	2.470619
Log likelihood	-206.2024	Hannan-Quinn criter.	2.459615
Durbin-Watson stat	1.644728		

Figure 22: EQ03: ARMA(0,0) model for ΔY_t.

Dependent Variable: D(Y)
Method: Least Squares
Date: 04/25/19 Time: 16:53
Sample (adjusted): 1960Q2 2002Q2
Included observations: 169 after adjustments
Convergence achieved after 8 iterations
MA Backcast: 1960Q1

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.013914	0.085804	-0.162162	0.8714
MA(1)	0.403445	0.070865	5.693162	0.0000

R-squared	0.068728	Mean dependent var	-0.013176
Adjusted R-squared	0.063152	S.D. dependent var	0.822145
S.E. of regression	0.795762	Akaike info criterion	2.392730
Sum squared resid	105.7505	Schwarz criterion	2.429770
Log likelihood	-200.1857	Hannan-Quinn criter.	2.407761
F-statistic	12.32462	Durbin-Watson stat	2.218397
Prob(F-statistic)	0.000575		

Inverted MA Roots	-.40

Figure 23: EQ04: ARMA(0,1) model for ΔY_t.

Dependent Variable: D(Y)
Method: Least Squares
Date: 04/25/19 Time: 16:54
Sample (adjusted): 1960Q2 2002Q2
Included observations: 169 after adjustments
Convergence achieved after 12 iterations
MA Backcast: 1959Q4 1960Q1

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.012715	0.064473	-0.197221	0.8439
MA(1)	0.293512	0.075920	3.866090	0.0002
MA(2)	-0.212582	0.075931	-2.799673	0.0057

R-squared	0.122726	Mean dependent var	-0.013176
Adjusted R-squared	0.112157	S.D. dependent var	0.822145
S.E. of regression	0.774670	Akaike info criterion	2.344832
Sum squared resid	99.61876	Schwarz criterion	2.400392
Log likelihood	-195.1383	Hannan-Quinn criter.	2.367379
F-statistic	11.61129	Durbin-Watson stat	2.003572
Prob(F-statistic)	0.000019		

Inverted MA Roots	.34	-.63

Figure 24: EQ05: ARMA(0,2) model for ΔY_t.

Dependent Variable: D(Y)
Method: Least Squares
Date: 04/25/19 Time: 16:55
Sample (adjusted): 1960Q3 2002Q2
Included observations: 168 after adjustments
Convergence achieved after 3 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.008170	0.076034	-0.107459	0.9146
AR(1)	0.175556	0.076245	2.302522	0.0225

R-squared	0.030949	Mean dependent var	-0.009087
Adjusted R-squared	0.025111	S.D. dependent var	0.822878
S.E. of regression	0.812480	Akaike info criterion	2.434384
Sum squared resid	109.5807	Schwarz criterion	2.471574
Log likelihood	-202.4882	Hannan-Quinn criter.	2.449477
F-statistic	5.301608	Durbin-Watson stat	1.918333
Prob(F-statistic)	0.022547		

Inverted AR Roots	.18

Figure 25: EQ06: ARMA(1,0) model for ΔY_t.

Dependent Variable: D(Y)
Method: Least Squares
Date: 04/25/19 Time: 16:55
Sample (adjusted): 1960Q3 2002Q2
Included observations: 168 after adjustments
Convergence achieved after 15 iterations
MA Backcast: 1960Q2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.010358	0.074892	-0.138308	0.8902
AR(1)	-0.348812	0.169965	-2.052263	0.0417
MA(1)	0.676648	0.133341	5.074559	0.0000

R-squared	0.108358	Mean dependent var	-0.009087
Adjusted R-squared	0.097550	S.D. dependent var	0.822878
S.E. of regression	0.781712	Akaike info criterion	2.363036
Sum squared resid	100.8272	Schwarz criterion	2.418821
Log likelihood	-195.4950	Hannan-Quinn criter.	2.385676
F-statistic	10.02593	Durbin-Watson stat	2.095406
Prob(F-statistic)	0.000078		

Inverted AR Roots	-.35
Inverted MA Roots	-.68

Figure 26: EQ07: ARMA(1,1) model for ΔY_t.

Dependent Variable: D(Y)
Method: Least Squares
Date: 04/25/19 Time: 16:56
Sample (adjusted): 1960Q3 2002Q2
Included observations: 168 after adjustments
Convergence achieved after 18 iterations
MA Backcast: 1960Q1 1960Q2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.009593	0.065132	-0.147287	0.8831
AR(1)	-0.032117	0.363522	-0.088350	0.9297
MA(1)	0.321024	0.356771	0.899804	0.3695
MA(2)	-0.202164	0.151291	-1.336260	0.1833

R-squared	0.121874	Mean dependent var	-0.009087
Adjusted R-squared	0.105811	S.D. dependent var	0.822878
S.E. of regression	0.778126	Akaike info criterion	2.359666
Sum squared resid	99.29885	Schwarz criterion	2.434046
Log likelihood	-194.2120	Hannan-Quinn criter.	2.389853
F-statistic	7.587106	Durbin-Watson stat	2.006587
Prob(F-statistic)	0.000088		

Inverted AR Roots	-.03	
Inverted MA Roots	.32	-.64

Figure 27: EQ08: ARMA(1,2) model for ΔY_t.

Dependent Variable: D(Y)
Method: Least Squares
Date: 04/25/19 Time: 16:57
Sample (adjusted): 1960Q4 2002Q2
Included observations: 167 after adjustments
Convergence achieved after 3 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.005609	0.059498	-0.094265	0.9250
AR(1)	0.214356	0.075567	2.836639	0.0051
AR(2)	-0.243233	0.075411	-3.225406	0.0015

R-squared	0.087456	Mean dependent var	-0.004371
Adjusted R-squared	0.076327	S.D. dependent var	0.823072
S.E. of regression	0.791038	Akaike info criterion	2.386858
Sum squared resid	102.6215	Schwarz criterion	2.442870
Log likelihood	-196.3027	Hannan-Quinn criter.	2.409592
F-statistic	7.858644	Durbin-Watson stat	1.886421
Prob(F-statistic)	0.000551		

Inverted AR Roots	.11+.48i	.11-.48i

Figure 28: EQ09: ARMA(2,0) model for ΔY_t.

Dependent Variable: D(Y)
Method: Least Squares
Date: 04/25/19 Time: 16:57
Sample (adjusted): 1960Q4 2002Q2
Included observations: 167 after adjustments
Convergence achieved after 14 iterations
MA Backcast: 1960Q3

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.007273	0.063268	-0.114949	0.9086
AR(1)	-0.259665	0.177386	-1.463844	0.1452
AR(2)	-0.195673	0.096088	-2.036402	0.0433
MA(1)	0.539483	0.173288	3.113223	0.0022

R-squared	0.132030	Mean dependent var	-0.004371
Adjusted R-squared	0.116055	S.D. dependent var	0.823072
S.E. of regression	0.773839	Akaike info criterion	2.348755
Sum squared resid	97.60880	Schwarz criterion	2.423438
Log likelihood	-192.1211	Hannan-Quinn criter.	2.379067
F-statistic	8.264821	Durbin-Watson stat	1.972602
Prob(F-statistic)	0.000038		

Inverted AR Roots	-.13+.42i	-.13-.42i
Inverted MA Roots	-.54	

Figure 29: EQ10: ARMA(2,1) model for ΔY_t.

Dependent Variable: D(Y)
Method: Least Squares
Date: 04/25/19 Time: 16:58
Sample (adjusted): 1960Q4 2002Q2
Included observations: 167 after adjustments
Convergence achieved after 23 iterations
MA Backcast: 1960Q2 1960Q3

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.006457	0.069115	-0.093420	0.9257
AR(1)	-1.042607	0.207530	-5.023878	0.0000
AR(2)	-0.537606	0.119335	-4.505015	0.0000
MA(1)	1.372371	0.210920	6.506582	0.0000
MA(2)	0.642513	0.162743	3.948028	0.0001

R-squared	0.157962	Mean dependent var	-0.004371
Adjusted R-squared	0.137170	S.D. dependent var	0.823072
S.E. of regression	0.764541	Akaike info criterion	2.330399
Sum squared resid	94.69261	Schwarz criterion	2.423753
Log likelihood	-189.5884	Hannan-Quinn criter.	2.368289
F-statistic	7.597565	Durbin-Watson stat	2.031619
Prob(F-statistic)	0.000012		

Inverted AR Roots	-.52-.52i	-.52+.52i
Inverted MA Roots	-.69-.41i	-.69+.41i

Figure 30: EQ11: ARMA(2,2) model for ΔY_t.

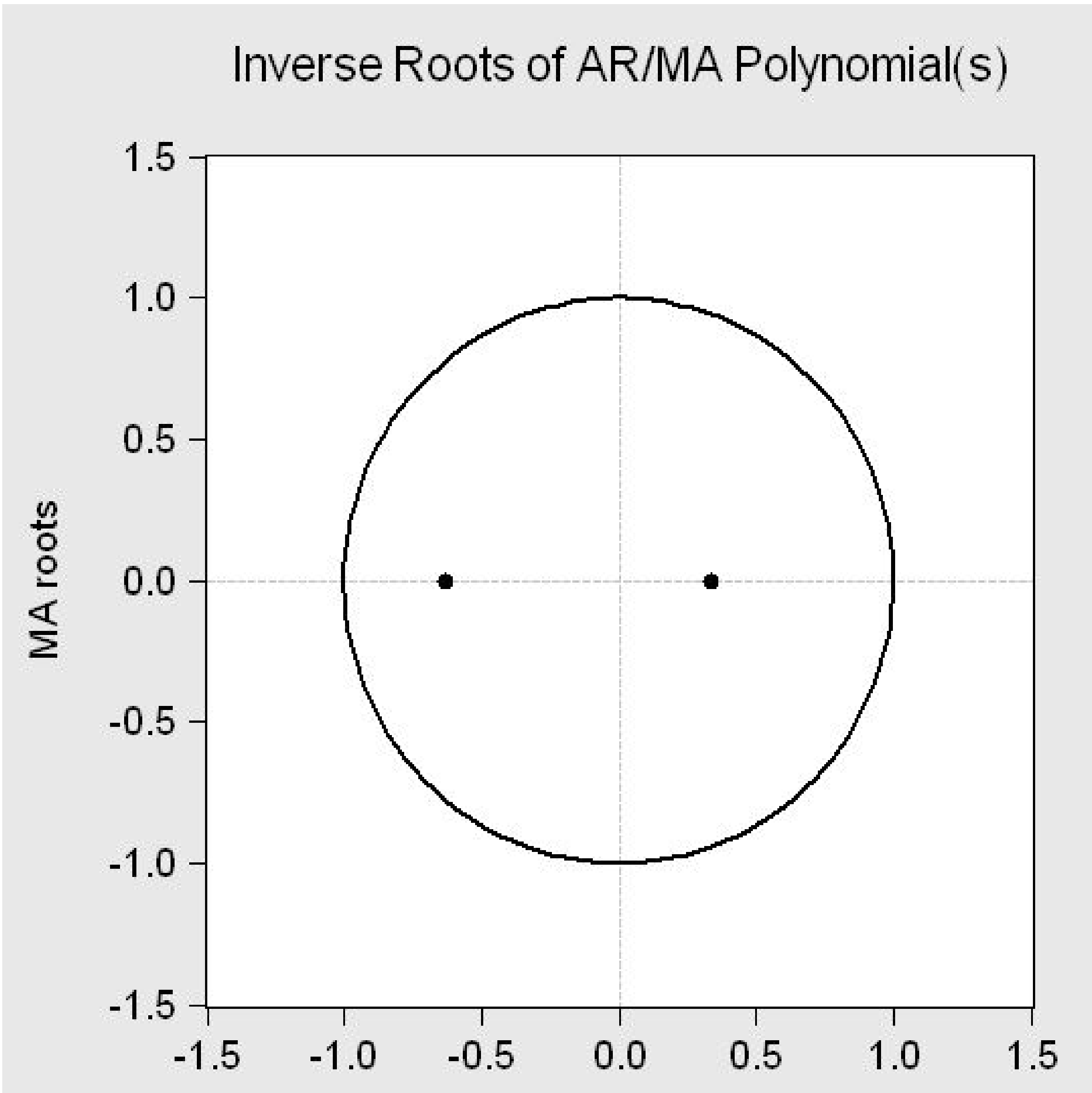


Figure 31: EQ05: ARMA(0,2) estimated inverse roots λ.

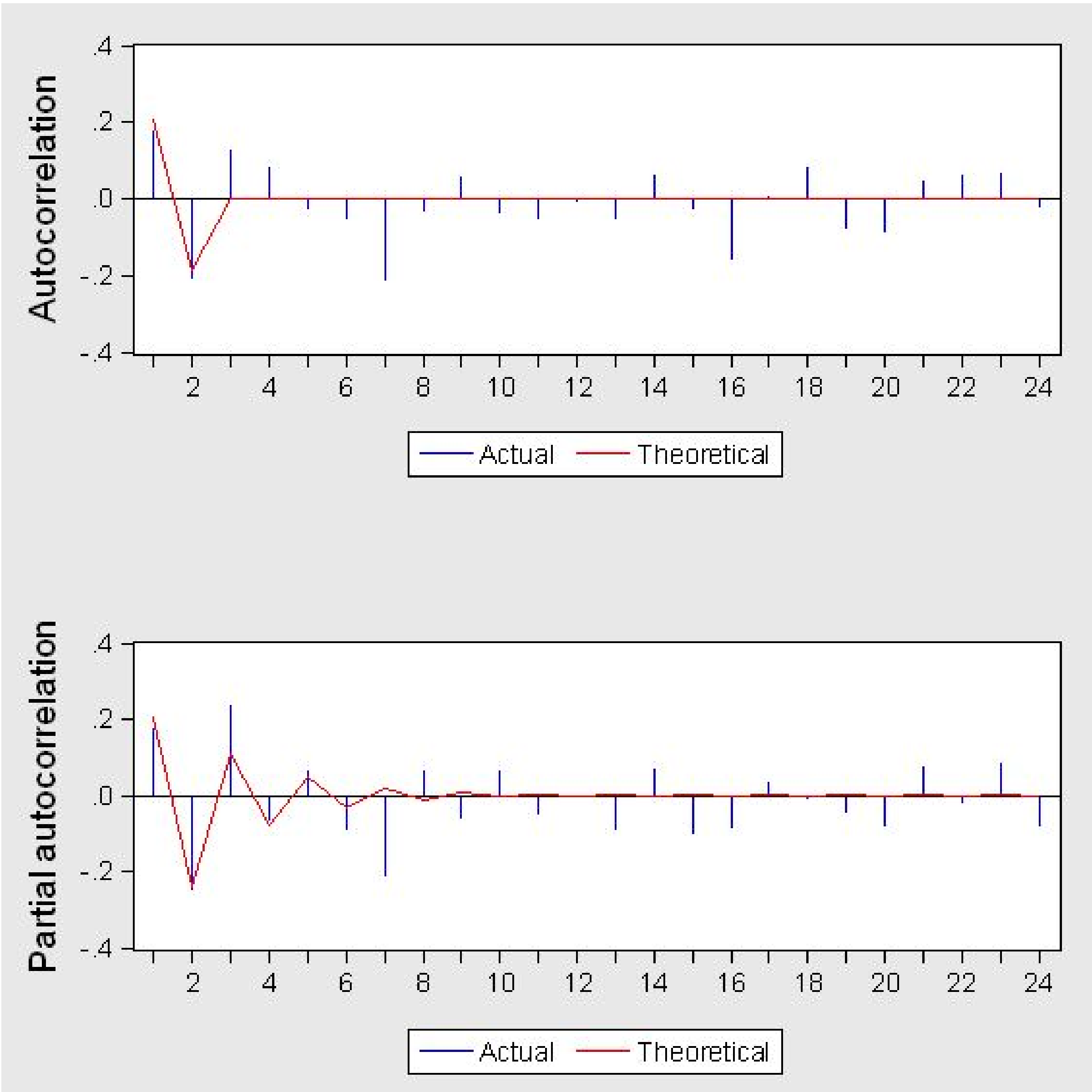


Figure 32: EQ05: ARMA(0,2) estimated correlograms.

Correlogram of Residuals

Date: 04/25/19 Time: 17:04
Sample: 1960Q2 2002Q2
Included observations: 169
Q-statistic probabilities adjusted for 2 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.004	-0.004	0.0032	
		2	-0.025	-0.025	0.1080	
		3	0.102	0.102	1.9086	0.167
		4	0.078	0.079	2.9699	0.227
		5	-0.072	-0.067	3.8757	0.275
		6	0.035	0.028	4.0971	0.393
		7	-0.238	-0.262	14.188	0.014
		8	0.024	0.037	14.293	0.027
		9	-0.005	-0.016	14.298	0.046
		10	-0.007	0.043	14.306	0.074
		11	-0.077	-0.039	15.379	0.081
		12	0.038	0.001	15.649	0.110
		13	-0.091	-0.085	17.174	0.103
		14	0.067	0.019	18.018	0.115
		15	-0.027	-0.012	18.150	0.152
		16	-0.128	-0.134	21.224	0.096
		17	0.002	0.027	21.225	0.130
		18	0.057	-0.001	21.844	0.148
		19	-0.065	-0.006	22.651	0.161
		20	-0.069	-0.109	23.587	0.169
		21	0.057	0.069	24.216	0.188
		22	0.022	0.001	24.308	0.229
		23	0.068	0.058	25.217	0.238
		24	-0.005	-0.018	25.222	0.287
		25	-0.041	-0.047	25.553	0.322
		26	0.100	0.083	27.589	0.278
		27	-0.036	-0.114	27.856	0.315
		28	-0.021	0.056	27.946	0.361
		29	-0.064	-0.132	28.796	0.371
		30	-0.033	0.029	29.021	0.411
		31	0.078	0.074	30.296	0.399
		32	0.040	0.021	30.628	0.434
		33	-0.091	-0.043	32.396	0.398
		34	0.022	-0.018	32.501	0.442
		35	-0.032	-0.064	32.719	0.481
		36	-0.032	-0.091	32.947	0.519

Figure 33: EQ05: ARMA(0,2) residual correlograms.

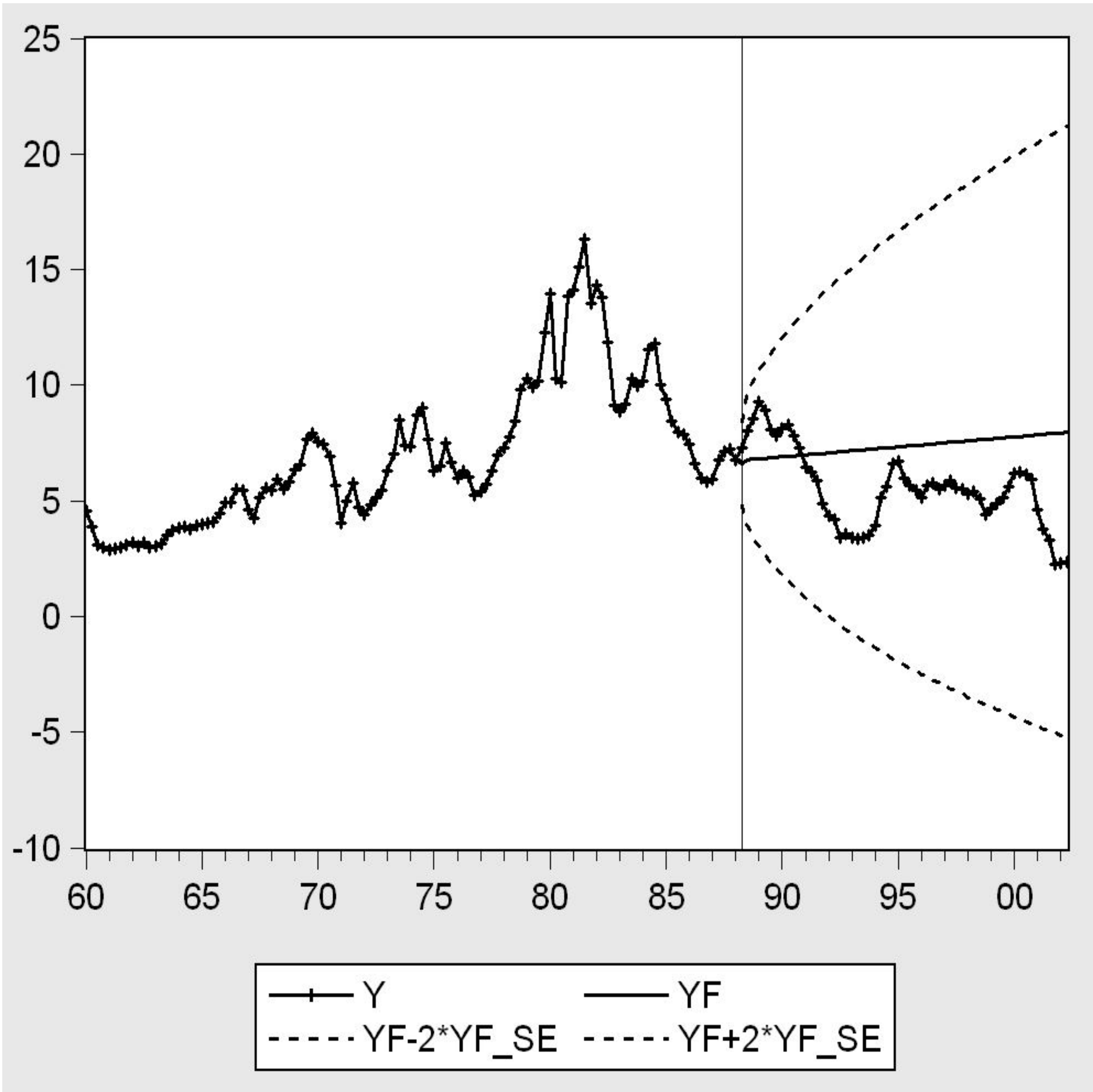


Figure 34: EQ05: ARMA(0,2) dynamic forecast (out-of-sample) for 1988Q2 to 2002Q2 (Y is the data, YF is the model forecast, and YF_SE is the forecast standard error).

Areas Under the Normal Curve

Z	Cum p	Tail p	Z	Cum p	Tail p	Z	Cum p	Tail p	Z	Cum p	Tail p	Z	Cum p	Tail p
0.00	0.5000	0.5000	0.40	0.6554	0.3446	0.80	0.7881	0.2119	1.20	0.8849	0.1151	1.60	0.9452	0.0548
0.01	0.5040	0.4960	0.41	0.6591	0.3409	0.81	0.7910	0.2090	1.21	0.8869	0.1131	1.61	0.9463	0.0537
0.02	0.5080	0.4920	0.42	0.6628	0.3372	0.82	0.7939	0.2061	1.22	0.8888	0.1112	1.62	0.9474	0.0526
0.03	0.5120	0.4880	0.43	0.6664	0.3336	0.83	0.7967	0.2033	1.23	0.8907	0.1093	1.63	0.9484	0.0516
0.04	0.5160	0.4840	0.44	0.6700	0.3300	0.84	0.7995	0.2005	1.24	0.8925	0.1075	1.64	0.9495	0.0505
0.05	0.5199	0.4801	0.45	0.6736	0.3264	0.85	0.8023	0.1977	1.25	0.8944	0.1056	**1.65**	**0.9505**	**0.0495**
0.06	0.5239	0.4761	0.46	0.6772	0.3228	0.86	0.8051	0.1949	1.26	0.8962	0.1038	1.66	0.9515	0.0485
0.07	0.5279	0.4721	0.47	0.6808	0.3192	0.87	0.8078	0.1922	1.27	0.8980	0.1020	1.67	0.9525	0.0475
0.08	0.5319	0.4681	0.48	0.6844	0.3156	0.88	0.8106	0.1894	1.28	0.8997	0.1003	1.68	0.9535	0.0465
0.09	0.5359	0.4641	0.49	0.6879	0.3121	0.89	0.8133	0.1867	1.29	0.9015	0.0985	1.69	0.9545	0.0455
0.10	0.5398	0.4602	0.50	0.6915	0.3085	0.90	0.8159	0.1841	1.30	0.9032	0.0968	1.70	0.9554	0.0446
0.11	0.5438	0.4562	0.51	0.6950	0.3050	0.91	0.8186	0.1814	1.31	0.9049	0.0951	1.71	0.9564	0.0436
0.12	0.5478	0.4522	0.52	0.6985	0.3015	0.92	0.8212	0.1788	1.32	0.9066	0.0934	1.72	0.9573	0.0427
0.13	0.5517	0.4483	0.53	0.7019	0.2981	0.93	0.8238	0.1762	1.33	0.9082	0.0918	1.73	0.9582	0.0418
0.14	0.5557	0.4443	0.54	0.7054	0.2946	0.94	0.8264	0.1736	1.34	0.9099	0.0901	1.74	0.9591	0.0409
0.15	0.5596	0.4404	0.55	0.7088	0.2912	0.95	0.8289	0.1711	1.35	0.9115	0.0885	1.75	0.9599	0.0401
0.16	0.5636	0.4364	0.56	0.7123	0.2877	0.96	0.8315	0.1685	1.36	0.9131	0.0869	1.76	0.9608	0.0392
0.17	0.5675	0.4325	0.57	0.7157	0.2843	0.97	0.8340	0.1660	1.37	0.9147	0.0853	1.77	0.9616	0.0384
0.18	0.5714	0.4286	0.58	0.7190	0.2810	0.98	0.8365	0.1635	1.38	0.9162	0.0838	1.78	0.9625	0.0375
0.19	0.5753	0.4247	0.59	0.7224	0.2776	0.99	0.8389	0.1611	1.39	0.9177	0.0823	1.79	0.9633	0.0367
0.20	0.5793	0.4207	0.60	0.7257	0.2743	1.00	0.8413	0.1587	1.40	0.9192	0.0808	1.80	0.9641	0.0359
0.21	0.5832	0.4168	0.61	0.7291	0.2709	1.01	0.8438	0.1562	1.41	0.9207	0.0793	1.81	0.9649	0.0351
0.22	0.5871	0.4129	0.62	0.7324	0.2676	1.02	0.8461	0.1539	1.42	0.9222	0.0778	1.82	0.9656	0.0344
0.23	0.5910	0.4090	0.63	0.7357	0.2643	1.03	0.8485	0.1515	1.43	0.9236	0.0764	1.83	0.9664	0.0336
0.24	0.5948	0.4052	0.64	0.7389	0.2611	1.04	0.8508	0.1492	1.44	0.9251	0.0749	1.84	0.9671	0.0329
0.25	0.5987	0.4013	0.65	0.7422	0.2578	1.05	0.8531	0.1469	1.45	0.9265	0.0735	1.85	0.9678	0.0322
0.26	0.6026	0.3974	0.66	0.7454	0.2546	1.06	0.8554	0.1446	1.46	0.9279	0.0721	1.86	0.9686	0.0314
0.27	0.6064	0.3936	0.67	0.7486	0.2514	1.07	0.8577	0.1423	1.47	0.9292	0.0708	1.87	0.9693	0.0307
0.28	0.6103	0.3897	0.68	0.7517	0.2483	1.08	0.8599	0.1401	1.48	0.9306	0.0694	1.88	0.9699	0.0301
0.29	0.6141	0.3859	0.69	0.7549	0.2451	1.09	0.8621	0.1379	1.49	0.9319	0.0681	1.89	0.9706	0.0294
0.30	0.6179	0.3821	0.70	0.7580	0.2420	1.10	0.8643	0.1357	1.50	0.9332	0.0668	1.90	0.9713	0.0287
0.31	0.6217	0.3783	0.71	0.7611	0.2389	1.11	0.8665	0.1335	1.51	0.9345	0.0655	1.91	0.9719	0.0281
0.32	0.6255	0.3745	0.72	0.7642	0.2358	1.12	0.8686	0.1314	1.52	0.9357	0.0643	1.92	0.9726	0.0274
0.33	0.6293	0.3707	0.73	0.7673	0.2327	1.13	0.8708	0.1292	1.53	0.9370	0.0630	1.93	0.9732	0.0268
0.34	0.6331	0.3669	0.74	0.7704	0.2296	1.14	0.8729	0.1271	1.54	0.9382	0.0618	1.94	0.9738	0.0262
0.35	0.6368	0.3632	0.75	0.7734	0.2266	1.15	0.8749	0.1251	1.55	0.9394	0.0606	1.95	0.9744	0.0256
0.36	0.6406	0.3594	0.76	0.7764	0.2236	1.16	0.8770	0.1230	1.56	0.9406	0.0594	**1.96**	**0.9750**	**0.0250**
0.37	0.6443	0.3557	0.77	0.7794	0.2206	1.17	0.8790	0.1210	1.57	0.9418	0.0582	1.97	0.9756	0.0244
0.38	0.6480	0.3520	0.78	0.7823	0.2177	1.18	0.8810	0.1190	1.58	0.9429	0.0571	1.98	0.9761	0.0239
0.39	0.6517	0.3483	0.79	0.7852	0.2148	1.19	0.8830	0.1170	1.59	0.9441	0.0559	1.99	0.9767	0.0233

Figure 35: Statistical table for N(0, 1).

Critical Values of the t Distribution

df	2-tailed testing			1-tailed testing		
	• •			• •		
	0.1	0.05	0.01	0.1	0.05	0.01
5	2.015	2.571	4.032	1.476	2.015	3.365
6	1.943	2.447	3.707	1.440	1.943	3.143
7	1.895	2.365	3.499	1.415	1.895	2.998
8	1.860	2.306	3.355	1.397	1.860	2.896
9	1.833	2.262	3.250	1.383	1.833	2.821
10	1.812	2.228	3.169	1.372	1.812	2.764
11	1.796	2.201	3.106	1.363	1.796	2.718
12	1.782	2.179	3.055	1.356	1.782	2.681
13	1.771	2.160	3.012	1.350	1.771	2.650
14	1.761	2.145	2.977	1.345	1.761	2.624
15	1.753	2.131	2.947	1.341	1.753	2.602
16	1.746	2.120	2.921	1.337	1.746	2.583
17	1.740	2.110	2.898	1.333	1.740	2.567
18	1.734	2.101	2.878	1.330	1.734	2.552
19	1.729	2.093	2.861	1.328	1.729	2.539
20	1.725	2.086	2.845	1.325	1.725	2.528
21	1.721	2.080	2.831	1.323	1.721	2.518
22	1.717	2.074	2.819	1.321	1.717	2.508
23	1.714	2.069	2.807	1.319	1.714	2.500
24	1.711	2.064	2.797	1.318	1.711	2.492
25	1.708	2.060	2.787	1.316	1.708	2.485
26	1.706	2.056	2.779	1.315	1.706	2.479
27	1.703	2.052	2.771	1.314	1.703	2.473
28	1.701	2.048	2.763	1.313	1.701	2.467
29	1.699	2.045	2.756	1.311	1.699	2.462
30	1.697	2.042	2.750	1.310	1.697	2.457
40	1.684	2.021	2.704	1.303	1.684	2.423
50	1.676	2.009	2.678	1.299	1.676	2.403
60	1.671	2.000	2.660	1.296	1.671	2.390
80	1.664	1.990	2.639	1.292	1.664	2.374
100	1.660	1.984	2.626	1.290	1.660	2.364
120	1.658	1.980	2.617	1.289	1.658	2.358
• •	1.645	1.960	2.576	1.282	1.645	2.327

Figure 36: Statistical table for Student's $t(r)$.

Critical Values of the F Distribution
(α = .05)

df	df between										
within	1	2	3	4	5	6	7	8	12	24	∞
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.68	4.53	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.00	3.84	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.57	3.41	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.28	3.12	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.07	2.90	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	2.91	2.74	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.79	2.61	2.41
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.69	2.51	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.60	2.42	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.53	2.35	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.48	2.29	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.42	2.24	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.38	2.19	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.34	2.15	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.31	2.11	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.28	2.08	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.25	2.05	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.23	2.03	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.20	2.01	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.18	1.98	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.16	1.96	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.15	1.95	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.13	1.93	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.12	1.91	1.66
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.10	1.90	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.09	1.89	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.00	1.79	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	1.92	1.70	1.39
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	1.88	1.65	1.33
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.85	1.63	1.28
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.83	1.61	1.26
∞	3.84	3.00	2.61	2.37	2.22	2.10	2.01	1.94	1.75	1.52	1.00

Figure 37: Statistical table for $F(m, n)$ at the 5% level.

Critical Values of the F Distribution
(α = .01)

df within	df between										
	1	2	3	4	5	6	7	8	12	24	∞
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	9.89	9.47	9.02
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.72	7.31	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.47	6.07	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.67	5.28	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.11	4.73	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.71	4.33	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.40	4.02	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.16	3.78	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	3.96	3.59	3.17
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	3.80	3.43	3.01
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.67	3.29	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.55	3.18	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.46	3.08	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.37	3.00	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.30	2.92	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.23	2.86	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.17	2.80	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.12	2.75	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.07	2.70	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.03	2.66	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	2.99	2.62	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	2.96	2.58	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	2.93	2.55	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	2.90	2.52	2.07
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	2.87	2.49	2.04
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	2.84	2.47	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.66	2.29	1.81
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.50	2.12	1.60
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.42	2.03	1.50
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.37	1.98	1.43
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.34	1.95	1.38
∞	6.64	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.19	1.79	1.00

Figure 38: Statistical table for $F(m, n)$ at the 1% level.

Critical Values of the χ^2 Distribution

df	Area in the Upper Tail					
	0.99	0.95	0.9	0.1	**0.05**	**0.01**
1	0.000	0.004	0.016	2.706	3.841	6.635
2	0.020	0.103	0.211	4.605	5.991	9.210
3	0.115	0.352	0.584	6.251	7.815	11.345
4	0.297	0.711	1.064	7.779	9.488	13.277
5	0.554	1.145	1.610	9.236	11.070	15.086
6	0.872	1.635	2.204	10.645	12.592	16.812
7	1.239	2.167	2.833	12.017	14.067	18.475
8	1.646	2.733	3.490	13.362	15.507	20.090
9	2.088	3.325	4.168	14.684	16.919	21.666
10	2.558	3.940	4.865	15.987	18.307	23.209
11	3.053	4.575	5.578	17.275	19.675	24.725
12	3.571	5.226	6.304	18.549	21.026	26.217
13	4.107	5.892	7.042	19.812	22.362	27.688
14	4.660	6.571	7.790	21.064	23.685	29.141
15	5.229	7.261	8.547	22.307	24.996	30.578
16	5.812	7.962	9.312	23.542	26.296	32.000
17	6.408	8.672	10.085	24.769	27.587	33.409
18	7.015	9.390	10.865	25.989	28.869	34.805
19	7.633	10.117	11.651	27.204	30.144	36.191
20	8.260	10.851	12.443	28.412	31.410	37.566
21	8.897	11.591	13.240	29.615	32.671	38.932
22	9.542	12.338	14.041	30.813	33.924	40.289
23	10.196	13.091	14.848	32.007	35.172	41.638
24	10.856	13.848	15.659	33.196	36.415	42.980
25	11.524	14.611	16.473	34.382	37.652	44.314

Figure 39: Statistical table for $\chi^2(q)$.