APS 2 solns

T = 595 binary: union (manual), occ (manual), ind (manufacturing), south, smsa (city), ms, fem, blk

other: ex, wks, ed, lwage

a

8 = 10 mins

ex 7–51 yrs mode 10–12.5 yrs median 21 yrs

$n/6\, SK^2 \sim \chi^2(1)$

" "

17.56 > 3.84 reject symmetry

ed 4–17 yrs mode 12 yrs (and 12+4 = 16 yrs) median 12 yrs

$n/6\, SK^2$ reject symmetry very few obsns 4–7 yrs

" 6.61

wks 5–52 wks outlier 5 wks mode 47.5–50 wks

median 48 wks reject symmetry

lwage 5.68 – 8.54 1982 prices

e.g. 2.92 5.00 (units? $/wk? => $15 000 – $265 000 /yr)

do not reject normality 95% => wage + skewed

binary: union (37%), occ (51%), ind (41%), south (29%),

218/595 smsa (64%), ms (18%), fem (11%), blk (7%)

→ 59% not manufacturing

bivariate correlations: union occ (manual) 39%

ed occ −64%

lwage occ −37%

lwage ms 32%

lwage fem −34%

lwage ed 46%

b

4 = 5 mins

$\hat{Prob}(union=1) = \Phi(2.52 - 0.0069\,ex - 0.061\,wks + 0.96\,occ - 0.093\,ind - 0.59\,south + 0.26\,smsa + 0.35\,ms - 0.41\,fem - 0.057\,ed + 0.23\,blk)$

(0.83)+++ (0.0056) (0.014)+++ (0.16)+++ (0.12) (0.14)+++ (0.13)++ (0.21)+ (0.20) (0.028)++ (0.094)

longest binary coeff.

cf. bivariate correlation smsa, union (0.8%, small)

significance of $\hat{\beta}$ wrt $f(x;\hat{\beta})\hat{\beta}$

residuals: substantial remaining structure (non-normal!)

c

10 = 12 mins

const	ex	wks	occ	ind	south	smsa	ms
2.52	−0.0069	−0.061	0.96	0.093	−0.59	0.26	0.35

fem	ed	blk
−0.41	−0.057	0.23

$\hat{\beta} = (2.52, -0.0069, -0.061, 0.96, 0.093, -0.59, 0.26, 0.35, -0.41, -0.057, 0.23)$

hard to interpret !

$\bar{x} = (1, 22.85, 46.45, \underbrace{0.51}_{occ}, \underbrace{0.41}_{ind}, \underbrace{0.29}_{south}, \underbrace{0.64}_{smsa}, \underbrace{0.81}_{ms}, \underbrace{0.11}_{fem}, 12.85, \underbrace{0.072}_{blk})$

m.e. $\left.\dfrac{\partial \widehat{Prob}(union=1)}{\partial x}\right|_{x=\bar{x}} = \phi(\bar{x}'\hat{\beta})\hat{\beta}$

$\bar{x}'\hat{\beta} = -0.43(022620011)$ PS

$\phi(\bar{x}'\hat{\beta}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(-0.43)^2} = 0.145(0866174)$ (−0.0925472916 3)

$\phi(\bar{x}'\hat{\beta})\hat{\beta}$ =		
0.36	const	
−0.0010	ex	(+ 10 yrs Prob ↓ 1%) note: insignificant $\hat{\beta}$ ⇏ insignificant m.e.
−0.0088	wks	+ 1 wk Prob ↓ 1%
0.14	occ	manual Prob ↑ 14%
0.013	ind	(manuf. Prob ↑ 1%)
−0.086	south	south Prob ↓ 9%
0.038	smsa	city Prob ↑ 4%
0.051	ms	married Prob ↑ 5%
−0.059	fem	(female Prob ↓ 6%)
−0.0083	ed	+ 1 yr Prob ↓ 1%
0.033	blk	(black Prob ↑ 3%)

quantitative

(d)

cannot ↑ 1 binary from $x=\bar{x}$

+ d

4 = 5 mins

e

8 = 10 mins

$H_0: \beta_6 = 0$ (smsa) $\quad H_1: \beta_6 \neq 0$ (2.05)

$t = \dfrac{\hat{\beta}_6}{\widehat{se}(\hat{\beta}_6)} = \dfrac{0.26}{0.13} \sim N(0,1)$ asymptotic (MLE nonlinear) 90% cv. ±1.65

Huber/White s.e.'s

reject H_0

$F = (R\hat{\beta}-r)'[R\,\widehat{Avar}(\hat{\beta})R']^{-1}(R\hat{\beta}-r) \sim \chi^2(1)$ $\quad q=1$

$t^2 = 4.18$ $\quad 2.71$ reject H_0 also at 95%

P-value 0.041

f

6 = 7 mins

$LR = -2(\ln \hat{L}_0 - \ln \hat{L}) \sim \chi^2(10)$ 99% cv $\chi^2_{0.99}(10) = 23.21$

155.08 $\quad -390.9177 \quad -313.3795 \quad k-1$ reject H_0

g

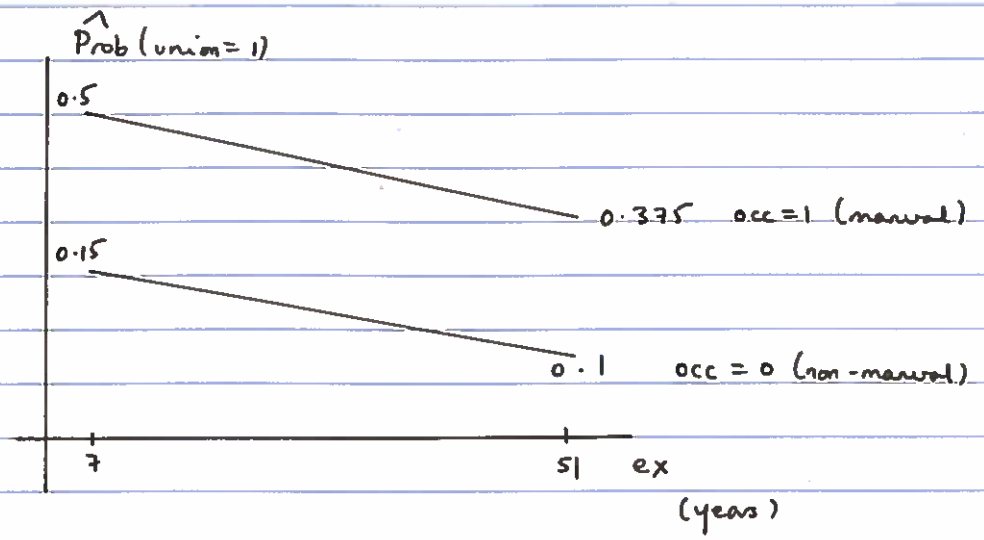
2 = 2 mins

$R^2_M = 1 - \dfrac{\ln \hat{L}}{\ln \hat{L}_0} = \dfrac{-313.3795}{-390.9177}$

0.20

h

8 = 10 mins



wks, ed — means // 46.5 wks; 12.8 yrs ed.

ind, south, smsa, ms — 1 } male, white, manufacturing, south, city, married

fem, blk — 0

assume that m.e's are significant

- $\widehat{Prob}$ (union = 1) ↓ in ex, for occ = 0, 1 (~⊖ 5-10%) over ex: 7-51

$\frac{\partial \widehat{Prob}(union=1)}{\partial occ}$ ~⊕ $0.\hat{\beta} - 0.3\hat{\beta}$ (↓ in ex) ($\hat{\beta}$, insignificant...)

0→1 / large (25-30%) slightly ~~unt~~ unsatisfactory.

↳ cf. part (d). me $(\bar{x})$ 0.14 ⇒ +14%.

k

actual

model	union = 0	union = 1	
$\widehat{union}$ = 0	314	86	400
$\widehat{union}$ = 1	63	132	195
	377 (83%)	218 (61%)	446/595 75%

$\Phi(.) \geq 0.5$ $\widehat{union} = 1$, else $\widehat{union} = 0$.

naïve: all $\widehat{union} = 0$ → global 63% correct ; cf R^2_M = 0.20

i $\widehat{Prob}(union=1) = \Lambda($ 4.38 (1.50)$^{+++}$ − 0.011 ex (0.0094) − 0.11 wks (0.026)$^{+++}$ + 1.66 occ (0.28)$^{+++}$

+ 0.18 ind (0.20) − 1.04 south (0.24)$^{+++}$ + 0.45 smsa (0.21)$^{++}$

+ 0.60 ms (0.36)$^{+}$ − 0.77 fem (0.54) − 0.091 ed (0.048)$^{+*}$ + 0.36 blk (0.42))

— qualitatively the same as (b).

$\bar{x}'\hat{\beta} = -0.73$ (093.017.2003)

$\frac{\hat{p}}{1-\hat{p}} = \widehat{OR}\big|_{x=\bar{x}} = e^{\bar{x}'\hat{\beta}} = 0.48$ (146094...)

$\hat{p} \simeq 0.48(1-\hat{p})$; $\hat{p}$ estd. probability that union = 1 is approx. half the estd. probability that union = 0.

j $\widehat{\log wage} = 5.59 + 0.057\,ed + 0.029\,ex - 0.00049\,ex^2$

$(0.19)^{+++}\quad (0.0066)^{+++}\quad (0.0065)^{+++}\quad (0.00013)^{+++}$

$+ 0.0034\,wks + 0.11\,union - 0.16\,occ + 0.085\,ind$ (+11%.)

$(0.0027)\quad (0.032)^{+++}\quad (0.037)^{+++}\quad (0.029)^{+++}$

$- 0.059\,south + 0.17\,smsa + 0.095\,ms - 0.32\,fem - 0.19\,blk$ (-32%. -19%.)

$(0.031)^{+}\quad (0.030)^{+++}\quad (0.049)^{+}\quad (0.061)^{+++}\quad (0.054)^{+++}$

$\hat{u}$: normality ok ; heteroscedasticity (White), autocorrelation (Breusch–Godfrey) borderline ok at 95% → standard OLS ses ($t \sim t(n-k)$, $F \sim F(q, n-k)$) ; Durbin–Watson $d = 1.86 \sim 2$

$R^2 = 46\%$.

(could test)

$\frac{\sqrt{T}}{2}(d-2) \sim N(0,1)$