
Topic: Discrete choice modelling (exam style)

- You are given cross-sectional data on the characteristics of $T = 595$ individuals from the 1982 U.S. Panel Study of Income Dynamics (PSID). The variables are: union membership (union= 1: union member, union= 0: not union member), years of full-time work experience (ex), weeks worked during year (wks), occupation (occ= 1: manual work, occ= 0: non-manual work), industry (ind= 1: manufacturing, ind= 0: non-manufacturing), residence (south= 1: southern U.S., south= 0: non-southern U.S.), residence (smsa= 1: city, smsa= 0: non-city), marital status (ms= 1: married, ms= 0: not married), sex (fem= 1: female, fem= 0: male), years of education (ed), race (blk= 1: black, blk= 0: non-black), and log wage (lwage).

- **Under exam conditions, questions (a) to (h) would be expected to take 60 minutes, for a total of 50 marks. For the in-class work, three additional questions have been added: (i), (j), and (k). In an exam, these would be awarded extra marks. Questions (j) and (k) require additional EViews manipulation, and this is not given in the handout.**

(a) Perform a brief preliminary analysis of the variables, and explain your findings, in no more than one paragraph. In addition: How many individuals are union members? What percentage of individuals are manual workers? What percentage of individuals are not in a manufacturing industry? What do you notice about the sample distribution of years of education?

(8 marks)

(b) The following probit equation has been estimated (probit_eqn):

$$\begin{aligned}\text{Prob}(\text{union} = 1) &= \Phi(\beta_0 + \beta_1\ \text{ex} + \beta_2\ \text{wks} + \beta_3\ \text{occ} + \beta_4\ \text{ind} + \beta_5\ \text{south} \\ &+ \beta_6\ \text{smsa} + \beta_7\ \text{ms} + \beta_8\ \text{fem} + \beta_9\ \text{ed} + \beta_{10}\ \text{blk}) + \text{error}\end{aligned}$$

What are the estimated coefficients? Briefly interpret the regression output.

(4 marks)

(c) Let x denote the vector of explanatory variables (including a constant), in the same order as listed in the probit model given in part (b) above. Compute the following:

$$\frac{\partial \widehat{\text{Prob}}(\text{union} = 1)}{\partial x} \quad \text{given} \quad \beta = \widehat{\beta}, x = \overline{x},$$

where $\widehat{\beta}$ are the estimated coefficients, and $\overline{x}$ is the sample mean of x.

(10 marks)

(d) Interpret your result on $\partial \widehat{\text{Prob}}(\text{union} = 1)/\partial x$ in part (c) above, for the explanatory variables occ (occupation) and ed (years of education).

(4 marks)

(e) Test $H_0 : \beta_6 = 0$ against $H_1 : \beta_6 \neq 0$, using t and Wald tests (perform the t test manually, and refer to the EViews output for the Wald test), at the 90% level. In each case, explain your choice of critical value.

(8 marks)

(f) Compute the likelihood-ratio (LR) statistic, and test the null that the restricted model is true (all coefficients except the constant are zero), against the alternative hypothesis that the null is not true, at the 99% level.

(6 marks)

(g) Compute McFadden's R^2 manually.

(2 marks)

(h) You are given probability response curves of $\widehat{\text{Prob}}(\text{union} = 1)$ against ex_plot (years of experience), for occ = 0 (non-manual worker) and occ = 1 (manual worker): these curves are labelled union_0 and union_1 respectively. The explanatory variables wks and ed are set to their sample means, while ind, south, smsa and ms are set to 1, and fem and blk are set to 0. Interpret the curves, with emphasis on the impact of occupation and years of experience.

(8 marks)

(i) Additional question for class exercise: Consider the logit model logit_eqn. Find the estimated odds-ratio evaluated at the sample mean, and interpret.

(0 marks)

(j) Additional question for class exercise: Estimate a model for the log wage, and interpret your results briefly.

(0 marks)

(k) Additional question for class exercise: Build a 2×2 table of hits and misses for probit_eqn, and use this to further assess the quality of the model.

(0 marks)

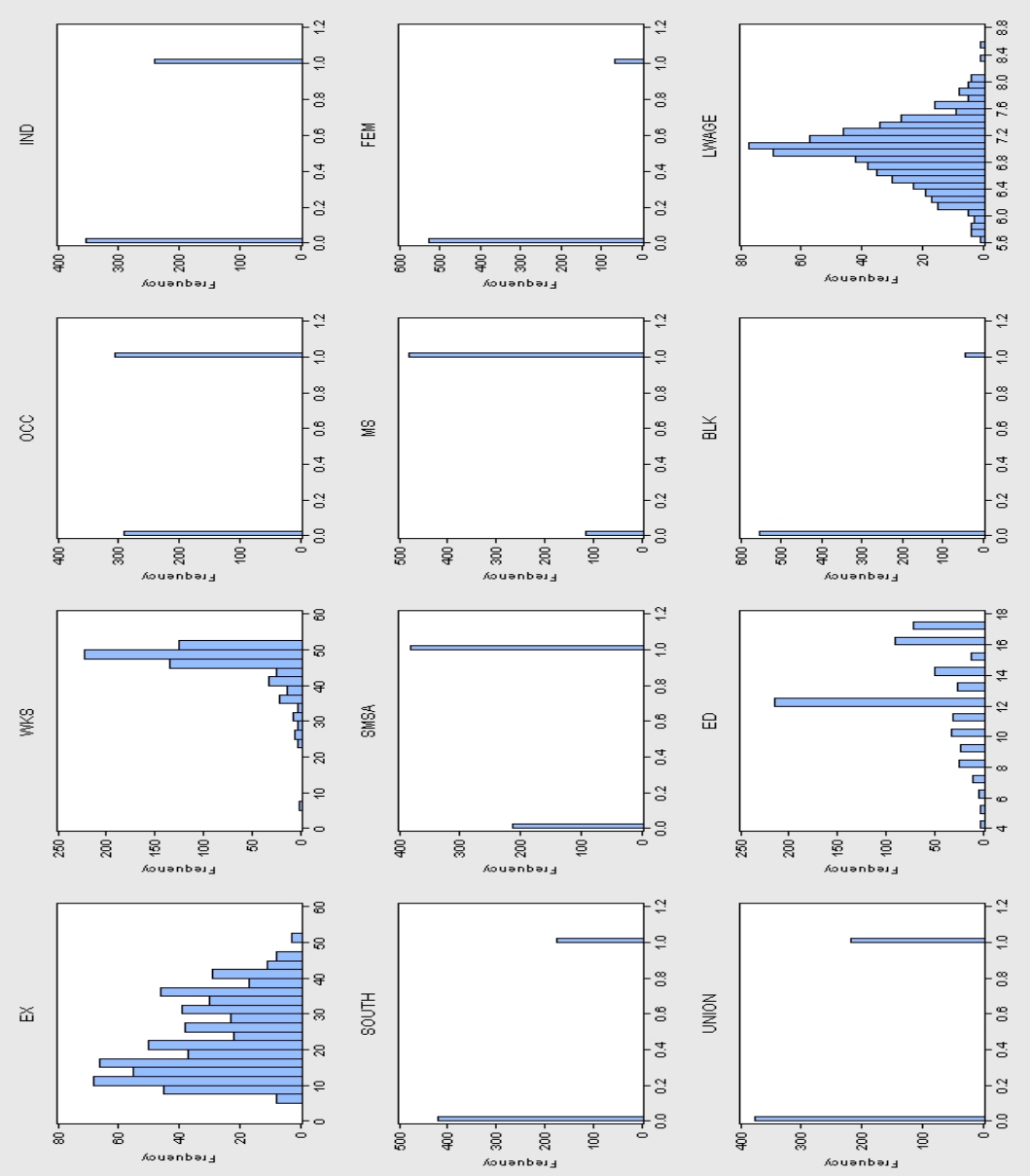


Figure 1: Histograms.

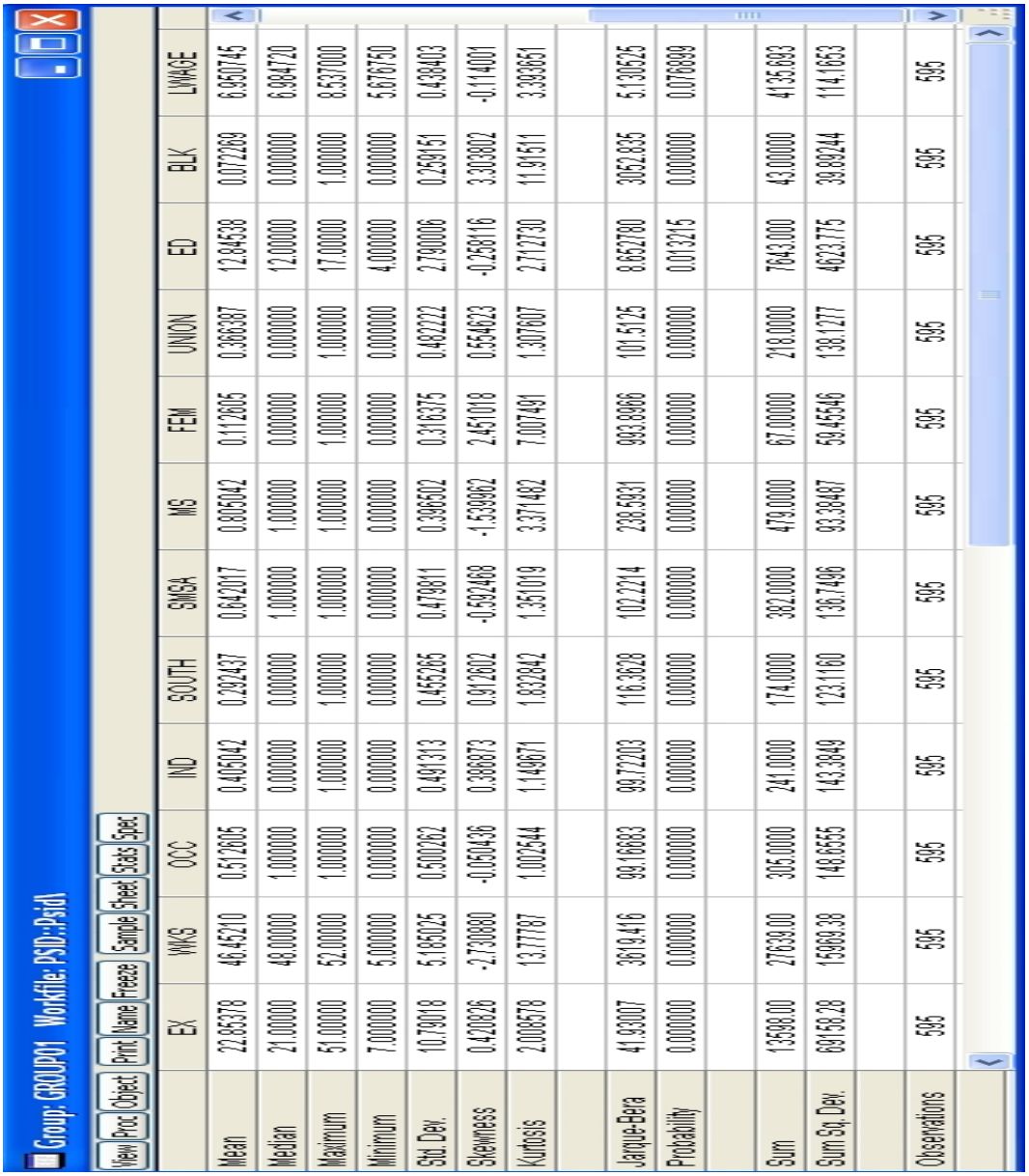


Figure 2: Descriptive statistics.

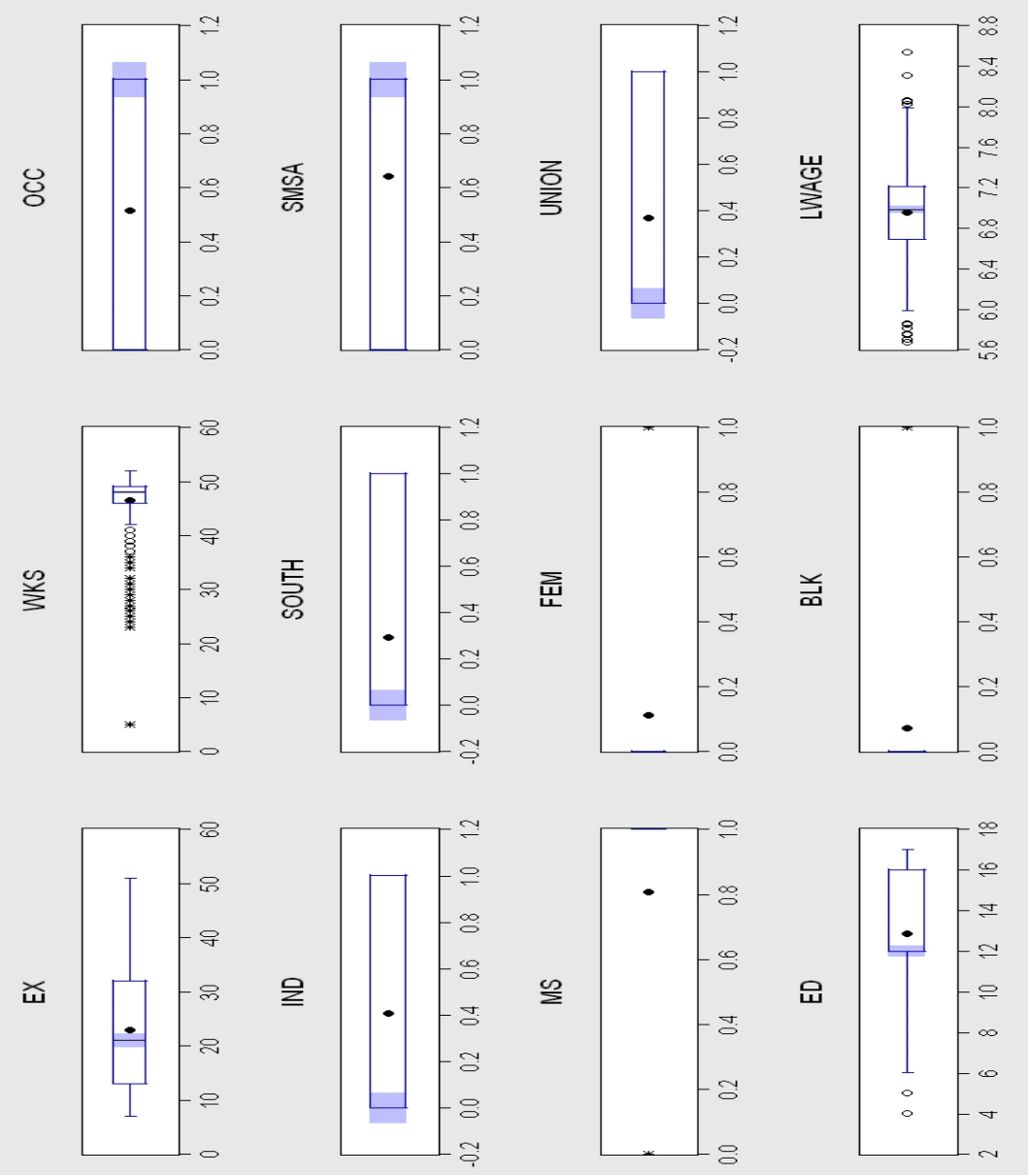
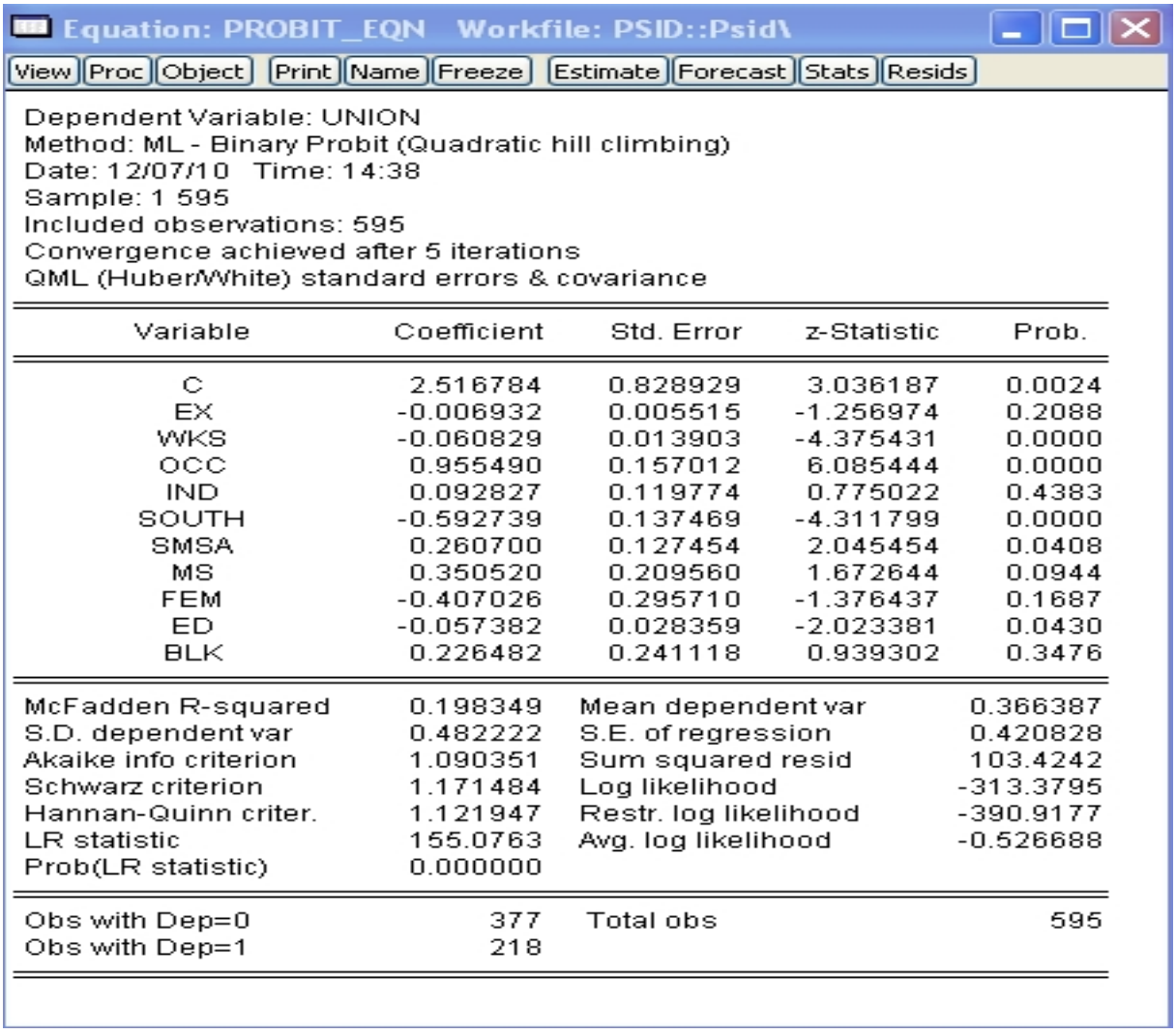


Figure 3: Boxplots.

Covariance Analysis: Ordinary
Date: 12/07/10 Time: 14:35
Sample: 1 595
Included observations: 595

Correlation	EX	WKS	OCC	IND	SOUTH	SMSA	MS	FEM	UNION	ED	BLK	LWAGE
EX	1.000000											
WKS	-0.106060	1.000000										
OCC	0.098116	-0.008367	1.000000									
IND	0.166477	-0.014510	0.249747	1.000000								
SOUTH	-0.057423	-0.031855	0.013355	-0.101437	1.000000							
SMSA	0.055558	0.021856	-0.188073	-0.040892	-0.144203	1.000000						
MS	0.157020	0.078156	0.029385	0.155424	-0.066004	-0.101995	1.000000					
FEM	-0.093799	-0.087531	-0.056849	-0.174782	0.063195	0.110735	-0.710447	1.000000				
UNION	0.068875	-0.172070	0.385583	0.168411	-0.159128	0.007570	0.118873	-0.127429	1.000000			
ED	-0.221922	0.000185	-0.641494	-0.247761	-0.110135	0.181175	0.018359	-0.001221	-0.271898	1.000000		
BLK	0.041112	-0.059437	0.077368	-0.045178	0.120220	0.127177	-0.223096	0.208577	0.030248	-0.119566	1.000000	
LWAGE	0.087326	0.040281	-0.366231	0.062364	-0.200633	0.242760	0.321815	-0.341863	0.018312	0.456567	-0.222913	1.000000

Figure 4: Correlations.



Equation: PROBIT_EQN Workfile: PSID::Psid\

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Dependent Variable: UNION
Method: ML - Binary Probit (Quadratic hill climbing)
Date: 12/07/10 Time: 14:38
Sample: 1 595
Included observations: 595
Convergence achieved after 5 iterations
QML (Huber/White) standard errors & covariance

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	2.516784	0.828929	3.036187	0.0024
EX	-0.006932	0.005515	-1.256974	0.2088
WKS	-0.060829	0.013903	-4.375431	0.0000
OCC	0.955490	0.157012	6.085444	0.0000
IND	0.092827	0.119774	0.775022	0.4383
SOUTH	-0.592739	0.137469	-4.311799	0.0000
SMSA	0.260700	0.127454	2.045454	0.0408
MS	0.350520	0.209560	1.672644	0.0944
FEM	-0.407026	0.295710	-1.376437	0.1687
ED	-0.057382	0.028359	-2.023381	0.0430
BLK	0.226482	0.241118	0.939302	0.3476

McFadden R-squared	0.198349	Mean dependent var	0.366387
S.D. dependent var	0.482222	S.E. of regression	0.420828
Akaike info criterion	1.090351	Sum squared resid	103.4242
Schwarz criterion	1.171484	Log likelihood	-313.3795
Hannan-Quinn criter.	1.121947	Restr. log likelihood	-390.9177
LR statistic	155.0763	Avg. log likelihood	-0.526688
Prob(LR statistic)	0.000000		

Obs with Dep=0	377	Total obs	595
Obs with Dep=1	218		

Figure 5: Regression results for probit_eqn.

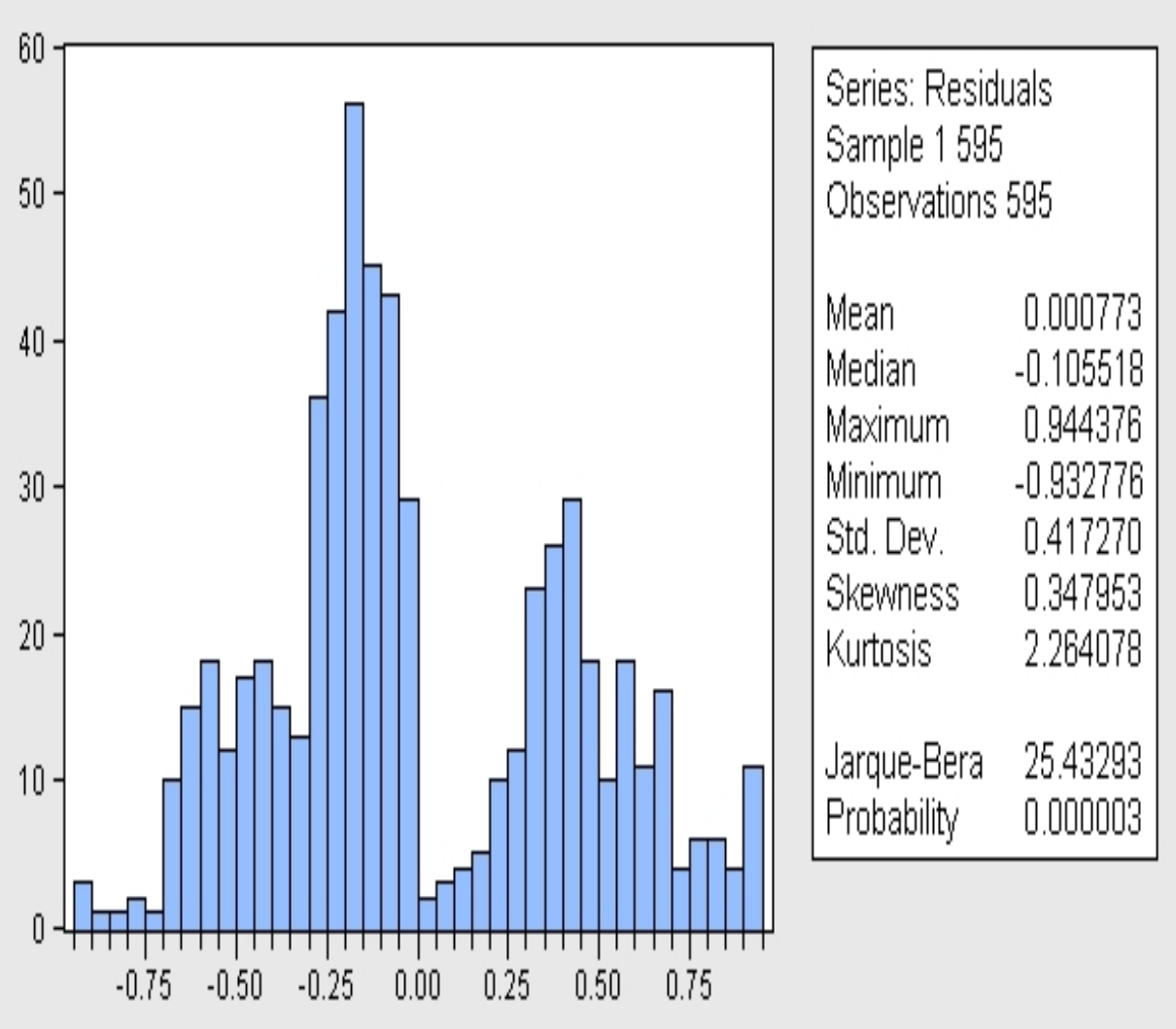
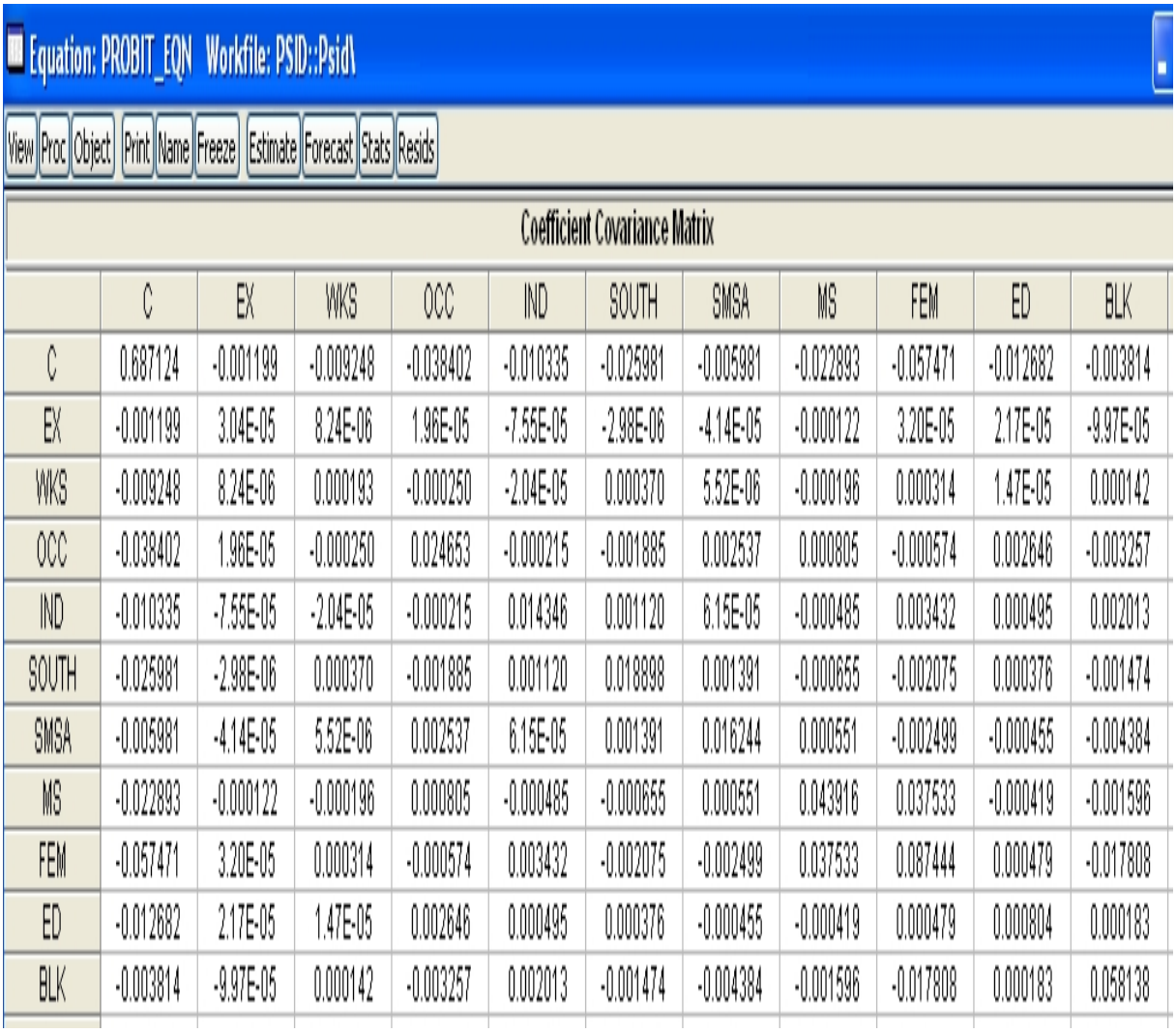


Figure 6: Estimated residuals from probit_eqn.



Equation: PROBIT_EQN Workfile: PSID::Psid\

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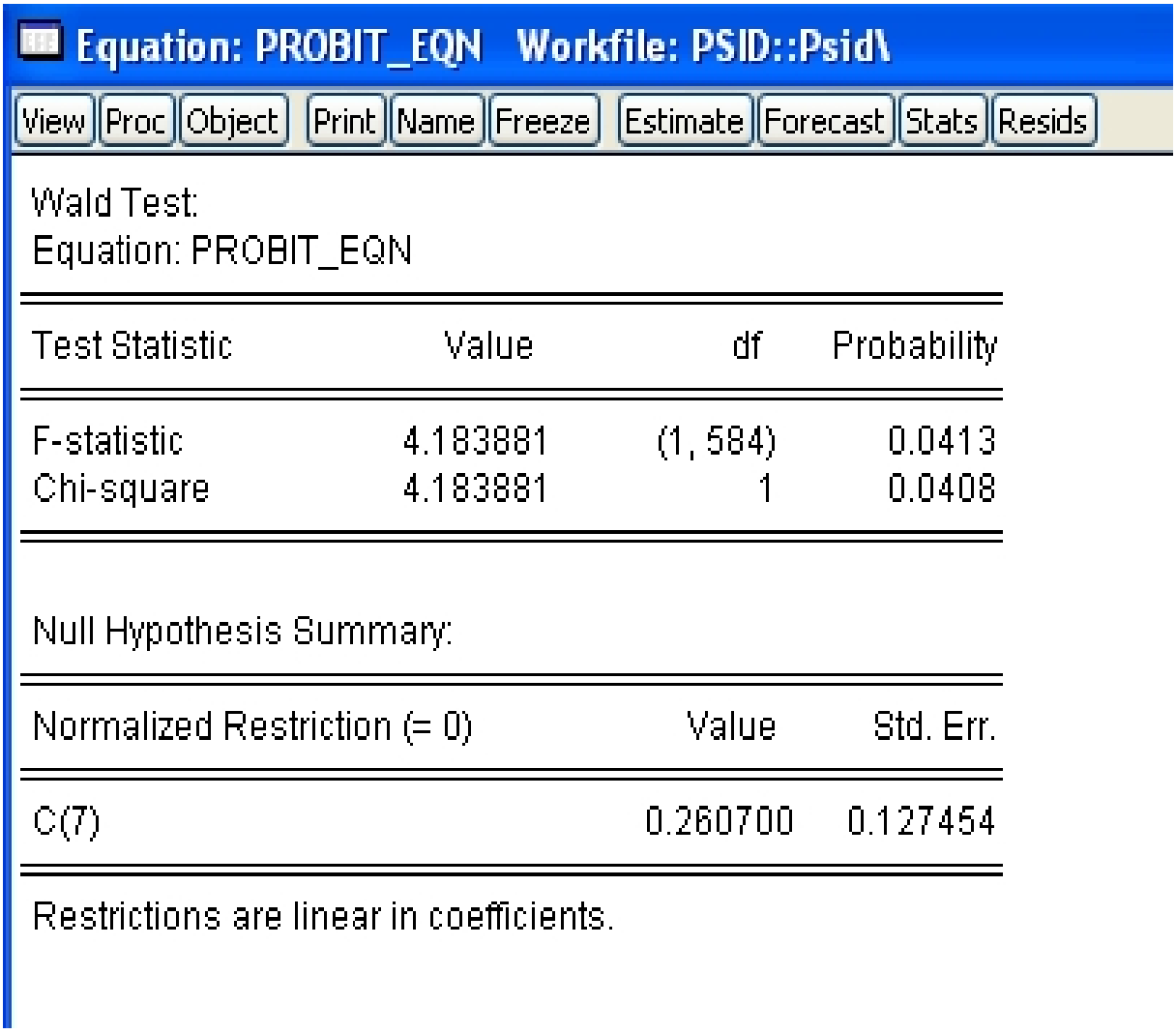
Coefficient Covariance Matrix

	C	EX	WKS	OCC	IND	SOUTH	SMSA	MS	FEM	ED	BLK
C	0.687124	-0.001199	-0.009248	-0.038402	-0.010335	-0.025981	-0.005981	-0.022893	-0.057471	-0.012682	-0.003814
EX	-0.001199	3.04E-05	8.24E-06	1.96E-05	-7.55E-05	-2.98E-06	-4.14E-05	-0.000122	3.20E-05	2.17E-05	-9.97E-05
WKS	-0.009248	8.24E-06	0.000193	-0.000250	-2.04E-05	0.000370	5.52E-06	-0.000196	0.000314	1.47E-05	0.000142
OCC	-0.038402	1.96E-05	-0.000250	0.024653	-0.000215	-0.001885	0.002537	0.000805	-0.000574	0.002646	-0.003257
IND	-0.010335	-7.55E-05	-2.04E-05	-0.000215	0.014346	0.001120	6.15E-05	-0.000485	0.003432	0.000495	0.002013
SOUTH	-0.025981	-2.98E-06	0.000370	-0.001885	0.001120	0.018898	0.001391	-0.000655	-0.002075	0.000376	-0.001474
SMSA	-0.005981	-4.14E-05	5.52E-06	0.002537	6.15E-05	0.001391	0.016244	0.000551	-0.002499	-0.000455	-0.004384
MS	-0.022893	-0.000122	-0.000196	0.000805	-0.000485	-0.000655	0.000551	0.043916	0.037533	-0.000419	-0.001596
FEM	-0.057471	3.20E-05	0.000314	-0.000574	0.003432	-0.002075	-0.002499	0.037533	0.087444	0.000479	-0.017808
ED	-0.012682	2.17E-05	1.47E-05	0.002646	0.000495	0.000376	-0.000455	-0.000419	0.000479	0.000804	0.000183
BLK	-0.003814	-9.97E-05	0.000142	-0.003257	0.002013	-0.001474	-0.004384	-0.001596	-0.017808	0.000183	0.058138

Figure 7: Estimated covariance matrix from probit_eqn.



Equation: PROBIT_EQN Workfile: PSID::Psid\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Wald Test:
Equation: PROBIT_EQN

Test Statistic	Value	df	Probability
F-statistic	4.183881	(1, 584)	0.0413
Chi-square	4.183881	1	0.0408

Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(7)	0.260700	0.127454

Restrictions are linear in coefficients.

Figure 8: Wald test output for probit_eqn.

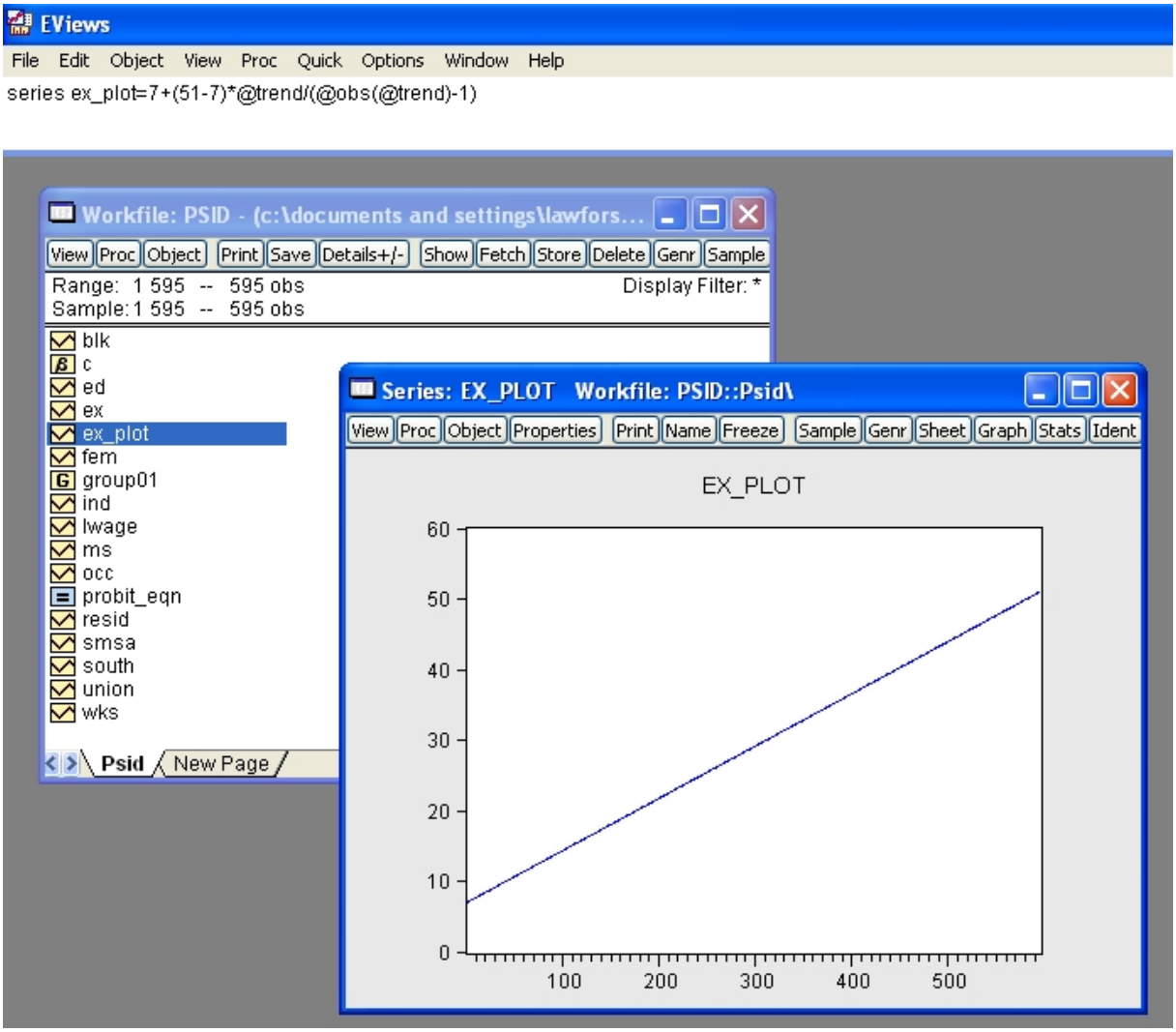


Figure 9: Variable definition: ex_plot.

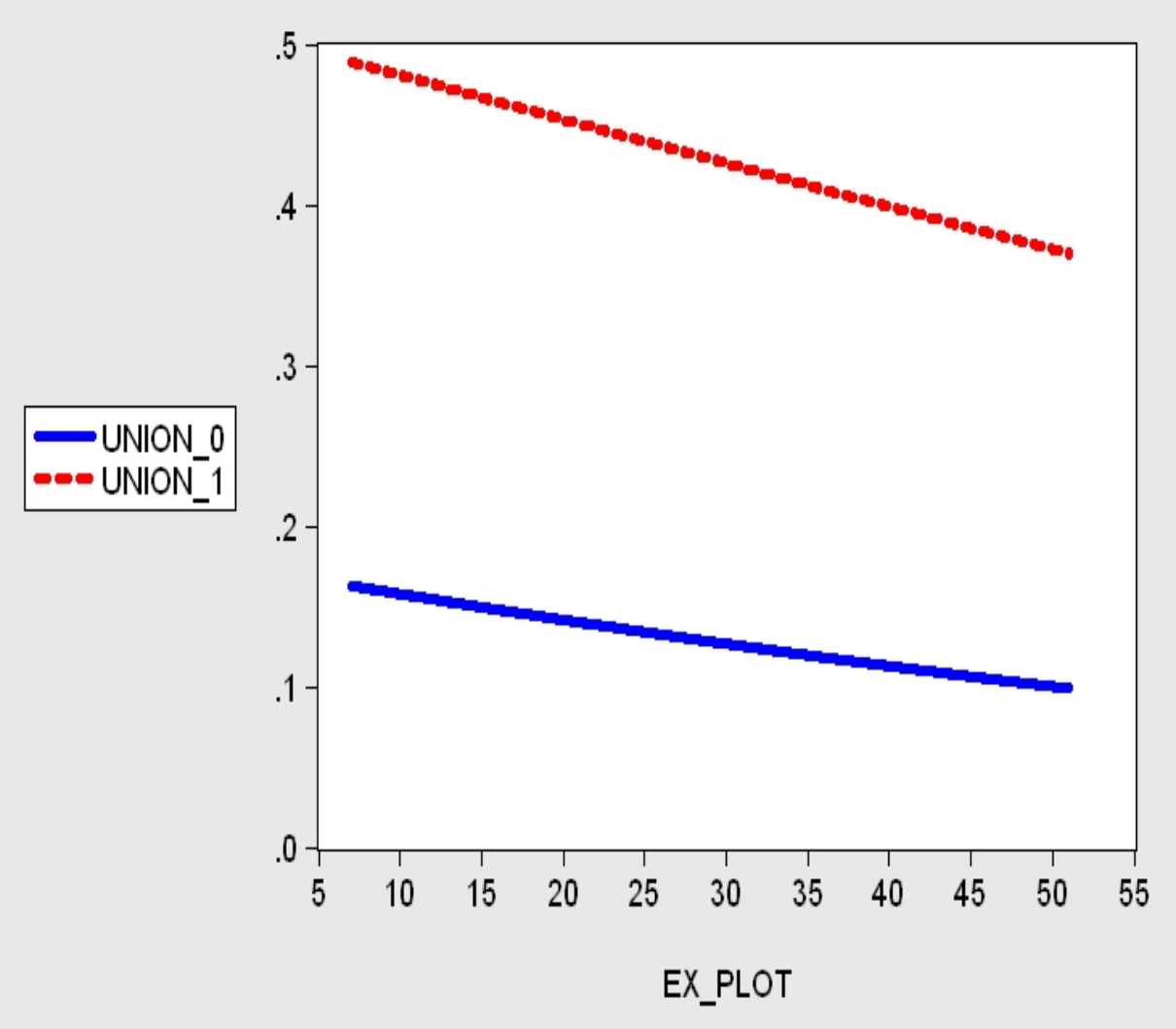


Figure 10: Probability response curves from probit_eqn.

Equation: LOGIT_EQN Workfile: PSID::Psid\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: UNION
Method: ML - Binary Logit (Quadratic hill climbing)
Date: 12/07/10 Time: 14:58
Sample: 1 595
Included observations: 595
Convergence achieved after 5 iterations
QML (Huber/White) standard errors & covariance

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	4.380828	1.500746	2.919099	0.0035
EX	-0.011143	0.009359	-1.190503	0.2338
WKS	-0.108126	0.025875	-4.178779	0.0000
OCC	1.658222	0.277676	5.971787	0.0000
IND	0.181818	0.200410	0.907232	0.3643
SOUTH	-1.044332	0.243292	-4.292500	0.0000
SMSA	0.448389	0.214601	2.089405	0.0367
MS	0.604999	0.360747	1.677074	0.0935
FEM	-0.772222	0.540173	-1.429583	0.1528
ED	-0.090799	0.048437	-1.874576	0.0609
BLK	0.355706	0.424359	0.838219	0.4019

McFadden R-squared	0.201017	Mean dependent var	0.366387
S.D. dependent var	0.482222	S.E. of regression	0.419664
Akaike info criterion	1.086846	Sum squared resid	102.8529
Schwarz criterion	1.167979	Log likelihood	-312.3367
Hannan-Quinn criter.	1.118442	Restr. log likelihood	-390.9177
LR statistic	157.1619	Avg. log likelihood	-0.524936
Prob(LR statistic)	0.000000		

Obs with Dep=0	377	Total obs	595
Obs with Dep=1	218		

Figure 11: Regression results for logit_eqn.

Areas Under the Normal Curve

Z	Cum p	Tail p	Z	Cum p	Tail p	Z	Cum p	Tail p	Z	Cum p	Tail p	Z	Cum p	Tail p
0.00	0.5000	0.5000	0.40	0.6554	0.3446	0.80	0.7881	0.2119	1.20	0.8849	0.1151	1.60	0.9452	0.0548
0.01	0.5040	0.4960	0.41	0.6591	0.3409	0.81	0.7910	0.2090	1.21	0.8869	0.1131	1.61	0.9463	0.0537
0.02	0.5080	0.4920	0.42	0.6628	0.3372	0.82	0.7939	0.2061	1.22	0.8888	0.1112	1.62	0.9474	0.0526
0.03	0.5120	0.4880	0.43	0.6664	0.3336	0.83	0.7967	0.2033	1.23	0.8907	0.1093	1.63	0.9484	0.0516
0.04	0.5160	0.4840	0.44	0.6700	0.3300	0.84	0.7995	0.2005	1.24	0.8925	0.1075	1.64	0.9495	0.0505
0.05	0.5199	0.4801	0.45	0.6736	0.3264	0.85	0.8023	0.1977	1.25	0.8944	0.1056	**1.65**	**0.9505**	**0.0495**
0.06	0.5239	0.4761	0.46	0.6772	0.3228	0.86	0.8051	0.1949	1.26	0.8962	0.1038	1.66	0.9515	0.0485
0.07	0.5279	0.4721	0.47	0.6808	0.3192	0.87	0.8078	0.1922	1.27	0.8980	0.1020	1.67	0.9525	0.0475
0.08	0.5319	0.4681	0.48	0.6844	0.3156	0.88	0.8106	0.1894	1.28	0.8997	0.1003	1.68	0.9535	0.0465
0.09	0.5359	0.4641	0.49	0.6879	0.3121	0.89	0.8133	0.1867	1.29	0.9015	0.0985	1.69	0.9545	0.0455
0.10	0.5398	0.4602	0.50	0.6915	0.3085	0.90	0.8159	0.1841	1.30	0.9032	0.0968	1.70	0.9554	0.0446
0.11	0.5438	0.4562	0.51	0.6950	0.3050	0.91	0.8186	0.1814	1.31	0.9049	0.0951	1.71	0.9564	0.0436
0.12	0.5478	0.4522	0.52	0.6985	0.3015	0.92	0.8212	0.1788	1.32	0.9066	0.0934	1.72	0.9573	0.0427
0.13	0.5517	0.4483	0.53	0.7019	0.2981	0.93	0.8238	0.1762	1.33	0.9082	0.0918	1.73	0.9582	0.0418
0.14	0.5557	0.4443	0.54	0.7054	0.2946	0.94	0.8264	0.1736	1.34	0.9099	0.0901	1.74	0.9591	0.0409
0.15	0.5596	0.4404	0.55	0.7088	0.2912	0.95	0.8289	0.1711	1.35	0.9115	0.0885	1.75	0.9599	0.0401
0.16	0.5636	0.4364	0.56	0.7123	0.2877	0.96	0.8315	0.1685	1.36	0.9131	0.0869	1.76	0.9608	0.0392
0.17	0.5675	0.4325	0.57	0.7157	0.2843	0.97	0.8340	0.1660	1.37	0.9147	0.0853	1.77	0.9616	0.0384
0.18	0.5714	0.4286	0.58	0.7190	0.2810	0.98	0.8365	0.1635	1.38	0.9162	0.0838	1.78	0.9625	0.0375
0.19	0.5753	0.4247	0.59	0.7224	0.2776	0.99	0.8389	0.1611	1.39	0.9177	0.0823	1.79	0.9633	0.0367
0.20	0.5793	0.4207	0.60	0.7257	0.2743	1.00	0.8413	0.1587	1.40	0.9192	0.0808	1.80	0.9641	0.0359
0.21	0.5832	0.4168	0.61	0.7291	0.2709	1.01	0.8438	0.1562	1.41	0.9207	0.0793	1.81	0.9649	0.0351
0.22	0.5871	0.4129	0.62	0.7324	0.2676	1.02	0.8461	0.1539	1.42	0.9222	0.0778	1.82	0.9656	0.0344
0.23	0.5910	0.4090	0.63	0.7357	0.2643	1.03	0.8485	0.1515	1.43	0.9236	0.0764	1.83	0.9664	0.0336
0.24	0.5948	0.4052	0.64	0.7389	0.2611	1.04	0.8508	0.1492	1.44	0.9251	0.0749	1.84	0.9671	0.0329
0.25	0.5987	0.4013	0.65	0.7422	0.2578	1.05	0.8531	0.1469	1.45	0.9265	0.0735	1.85	0.9678	0.0322
0.26	0.6026	0.3974	0.66	0.7454	0.2546	1.06	0.8554	0.1446	1.46	0.9279	0.0721	1.86	0.9686	0.0314
0.27	0.6064	0.3936	0.67	0.7486	0.2514	1.07	0.8577	0.1423	1.47	0.9292	0.0708	1.87	0.9693	0.0307
0.28	0.6103	0.3897	0.68	0.7517	0.2483	1.08	0.8599	0.1401	1.48	0.9306	0.0694	1.88	0.9699	0.0301
0.29	0.6141	0.3859	0.69	0.7549	0.2451	1.09	0.8621	0.1379	1.49	0.9319	0.0681	1.89	0.9706	0.0294
0.30	0.6179	0.3821	0.70	0.7580	0.2420	1.10	0.8643	0.1357	1.50	0.9332	0.0668	1.90	0.9713	0.0287
0.31	0.6217	0.3783	0.71	0.7611	0.2389	1.11	0.8665	0.1335	1.51	0.9345	0.0655	1.91	0.9719	0.0281
0.32	0.6255	0.3745	0.72	0.7642	0.2358	1.12	0.8686	0.1314	1.52	0.9357	0.0643	1.92	0.9726	0.0274
0.33	0.6293	0.3707	0.73	0.7673	0.2327	1.13	0.8708	0.1292	1.53	0.9370	0.0630	1.93	0.9732	0.0268
0.34	0.6331	0.3669	0.74	0.7704	0.2296	1.14	0.8729	0.1271	1.54	0.9382	0.0618	1.94	0.9738	0.0262
0.35	0.6368	0.3632	0.75	0.7734	0.2266	1.15	0.8749	0.1251	1.55	0.9394	0.0606	1.95	0.9744	0.0256
0.36	0.6406	0.3594	0.76	0.7764	0.2236	1.16	0.8770	0.1230	1.56	0.9406	0.0594	**1.96**	**0.9750**	**0.0250**
0.37	0.6443	0.3557	0.77	0.7794	0.2206	1.17	0.8790	0.1210	1.57	0.9418	0.0582	1.97	0.9756	0.0244
0.38	0.6480	0.3520	0.78	0.7823	0.2177	1.18	0.8810	0.1190	1.58	0.9429	0.0571	1.98	0.9761	0.0239
0.39	0.6517	0.3483	0.79	0.7852	0.2148	1.19	0.8830	0.1170	1.59	0.9441	0.0559	1.99	0.9767	0.0233

Figure 12: Statistical table for $N(0, 1)$. These tables are taken from `http://fsweb.berry.edu/academic/education/vbissonnette/tables/tables.html`.

Critical Values of the t Distribution

df	2-tailed testing			1-tailed testing		
	0.1	0.05	0.01	0.1	0.05	0.01
5	2.015	2.571	4.032	1.476	2.015	3.365
6	1.943	2.447	3.707	1.440	1.943	3.143
7	1.895	2.365	3.499	1.415	1.895	2.998
8	1.860	2.306	3.355	1.397	1.860	2.896
9	1.833	2.262	3.250	1.383	1.833	2.821
10	1.812	2.228	3.169	1.372	1.812	2.764
11	1.796	2.201	3.106	1.363	1.796	2.718
12	1.782	2.179	3.055	1.356	1.782	2.681
13	1.771	2.160	3.012	1.350	1.771	2.650
14	1.761	2.145	2.977	1.345	1.761	2.624
15	1.753	2.131	2.947	1.341	1.753	2.602
16	1.746	2.120	2.921	1.337	1.746	2.583
17	1.740	2.110	2.898	1.333	1.740	2.567
18	1.734	2.101	2.878	1.330	1.734	2.552
19	1.729	2.093	2.861	1.328	1.729	2.539
20	1.725	2.086	2.845	1.325	1.725	2.528
21	1.721	2.080	2.831	1.323	1.721	2.518
22	1.717	2.074	2.819	1.321	1.717	2.508
23	1.714	2.069	2.807	1.319	1.714	2.500
24	1.711	2.064	2.797	1.318	1.711	2.492
25	1.708	2.060	2.787	1.316	1.708	2.485
26	1.706	2.056	2.779	1.315	1.706	2.479
27	1.703	2.052	2.771	1.314	1.703	2.473
28	1.701	2.048	2.763	1.313	1.701	2.467
29	1.699	2.045	2.756	1.311	1.699	2.462
30	1.697	2.042	2.750	1.310	1.697	2.457
40	1.684	2.021	2.704	1.303	1.684	2.423
50	1.676	2.009	2.678	1.299	1.676	2.403
60	1.671	2.000	2.660	1.296	1.671	2.390
80	1.664	1.990	2.639	1.292	1.664	2.374
100	1.660	1.984	2.626	1.290	1.660	2.364
120	1.658	1.980	2.617	1.289	1.658	2.358
∞	1.645	1.960	2.576	1.282	1.645	2.327

Figure 13: Statistical table for Student's $t(r)$.

Critical Values of the F Distribution
($\alpha = .05$)

df within	df between										
	1	2	3	4	5	6	7	8	12	24	∞
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.68	4.53	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.00	3.84	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.57	3.41	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.28	3.12	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.07	2.90	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	2.91	2.74	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.79	2.61	2.41
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.69	2.51	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.60	2.42	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.53	2.35	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.48	2.29	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.42	2.24	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.38	2.19	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.34	2.15	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.31	2.11	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.28	2.08	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.25	2.05	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.23	2.03	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.20	2.01	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.18	1.98	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.16	1.96	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.15	1.95	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.13	1.93	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.12	1.91	1.66
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.10	1.90	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.09	1.89	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.00	1.79	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	1.92	1.70	1.39
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	1.88	1.65	1.33
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.85	1.63	1.28
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.83	1.61	1.26
∞	3.84	3.00	2.61	2.37	2.22	2.10	2.01	1.94	1.75	1.52	1.00

Figure 14: Statistical table for $F(m, n)$ at the 5% level.

Critical Values of the F Distribution
($\alpha = .01$)

df within	df between										
	1	2	3	4	5	6	7	8	12	24	∞
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	9.89	9.47	9.02
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.72	7.31	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.47	6.07	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.67	5.28	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.11	4.73	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.71	4.33	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.40	4.02	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.16	3.78	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	3.96	3.59	3.17
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	3.80	3.43	3.01
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.67	3.29	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.55	3.18	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.46	3.08	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.37	3.00	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.30	2.92	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.23	2.86	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.17	2.80	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.12	2.75	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.07	2.70	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.03	2.66	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	2.99	2.62	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	2.96	2.58	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	2.93	2.55	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	2.90	2.52	2.07
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	2.87	2.49	2.04
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	2.84	2.47	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.66	2.29	1.81
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.50	2.12	1.60
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.42	2.03	1.50
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.37	1.98	1.43
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.34	1.95	1.38
∞	6.64	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.19	1.79	1.00

Figure 15: Statistical table for $F(m, n)$ at the 1% level.

Critical Values of the χ^2 Distribution

df	Area in the Upper Tail					
	0.99	0.95	0.9	0.1	**0.05**	**0.01**
1	0.000	0.004	0.016	2.706	3.841	6.635
2	0.020	0.103	0.211	4.605	5.991	9.210
3	0.115	0.352	0.584	6.251	7.815	11.345
4	0.297	0.711	1.064	7.779	9.488	13.277
5	0.554	1.145	1.610	9.236	11.070	15.086
6	0.872	1.635	2.204	10.645	12.592	16.812
7	1.239	2.167	2.833	12.017	14.067	18.475
8	1.646	2.733	3.490	13.362	15.507	20.090
9	2.088	3.325	4.168	14.684	16.919	21.666
10	2.558	3.940	4.865	15.987	18.307	23.209
11	3.053	4.575	5.578	17.275	19.675	24.725
12	3.571	5.226	6.304	18.549	21.026	26.217
13	4.107	5.892	7.042	19.812	22.362	27.688
14	4.660	6.571	7.790	21.064	23.685	29.141
15	5.229	7.261	8.547	22.307	24.996	30.578
16	5.812	7.962	9.312	23.542	26.296	32.000
17	6.408	8.672	10.085	24.769	27.587	33.409
18	7.015	9.390	10.865	25.989	28.869	34.805
19	7.633	10.117	11.651	27.204	30.144	36.191
20	8.260	10.851	12.443	28.412	31.410	37.566
21	8.897	11.591	13.240	29.615	32.671	38.932
22	9.542	12.338	14.041	30.813	33.924	40.289
23	10.196	13.091	14.848	32.007	35.172	41.638
24	10.856	13.848	15.659	33.196	36.415	42.980
25	11.524	14.611	16.473	34.382	37.652	44.314

Figure 16: Statistical table for $\chi^2(q)$.