IENAC15/ Econometrics 2 Final Examination (RESIT)

Date: Thursday 27 October 2017.

Time allowed: 2 hours (13:15-15:15).

Answer all questions briefly.

Show all computations (including relevant critical values).

You are allowed all lecture handouts and notes, but no textbooks.

An English–other language dictionary is allowed, as is a scientific calculator.

Question 1 is for 100 marks.

You are given a "pooled" cross-sectional dataset with 1129 observations. It is taken from the U.S. National Opinion Research Center's *General Social Survey* for the even years from 1972 to 1984 inclusive. Each of the years in the dataset corresponds to a different cross-section: data on the same variables was recorded for a randomly chosen sample of individuals (females), and so **different** individuals were interviewed over time. The variables are AGE (age in years), AGESQ (age squared), BLACK (= 1 if black), EAST (= 1 if lived in eastern area at age 16), EDUC (years of schooling), FARM (= 1 if lived on a farm at age 16), FEDUC (father's level of education, in years), KIDS (number of children born to the individual), MEDUC (mother's level of education, in years), NORTHCEN (= 1 if lived in north-central area at age 16), OTHRURAL (= 1 if lived in non-farm rural area at age 16), SMCITY (= 1 if lived in a small city at age 16), TOWN (= 1 if lived in a town at age 16), WEST (= 1 if lived in western area at age 16), YEAR (year 1972 to 1984, even years only), Y74 (= 1 if YEAR = 1974), Y76 (= 1 if YEAR = 1976), Y78 (= 1 if YEAR = 1978), Y80 (= 1 if YEAR = 1980), Y82 (= 1 if YEAR = 1982), and Y84 (= 1 if YEAR = 1984). Interactions between the year dummies and EDUC are denoted by, e.g., Y74EDUC = Y74 × EDUC. **This is used in Question 1.**

1 Question 1

- This question uses the *General Social Survey* data (refer to Figures 1–15). We will use this data to construct models to explain the total number of children born to a given female (KIDS). One question of interest will be the following: after controlling for other observable factors, what has happened to fertility rates over time?

(a) Perform a careful first analysis of the variables, and explain your findings.

(10 marks)

(b) Discuss the output from EQ01 (the base year is 1972). Which (if any) of the classical assumptions appear to fail, and what are the consequences?

(20 marks)

(c) With reference to EQ02, what do the time dummies tell you about fertility? "Holding EDUC, AGE and the other factors fixed, 100 women in 1982 are predicted to have x fewer children than 100 comparable women in 1972": find x. (This drop is separate from the decline in fertility that is due to the increase in average education levels. The coefficients on the time dummies represent changes in fertility over time for reasons that are not captured in the explanatory variables). For additional evidence, find the mean level of education in 1972 and 1984.

(10 marks)

(d) While examining the EQ02 output, someone claims that, if everything else is equal, a black woman is expected to have one more child than a nonblack woman. Explain, with justification, whether you agree with this claim.

(10 marks)

(e) Given that some of the time dummies in EQ02 are individually quite significant, check whether as a group the year dummies are significant. Carefully explain. In light of your answer, discuss the results of EQ03.

(5 marks)

(f) From EQ02, we see that women with more education have fewer children. "Other things being equal, 100 women with a college education (4 years additional education over a non-college individual) will have about y fewer children on average than 100 women with no college education." Find y.

(5 marks)

(g) From EQ02, discuss the impact of age on fertility. Find any maxima or minima in the estimated quadratic. Discuss.

(10 marks)

(h) The model estimated in EQ02 assumes that the effect of each explanatory variable, particularly education, has remained constant over time. Discuss this carefully, using any appropriate supporting evidence that you can find or calculate.

(10 marks)

(i) In one paragraph, and using non-technical language, summarize your findings from parts (a)–(h) above, and discuss model improvements. You should assume that your audience is technically skilled, but has little working knowledge of econometrics.

(20 marks)

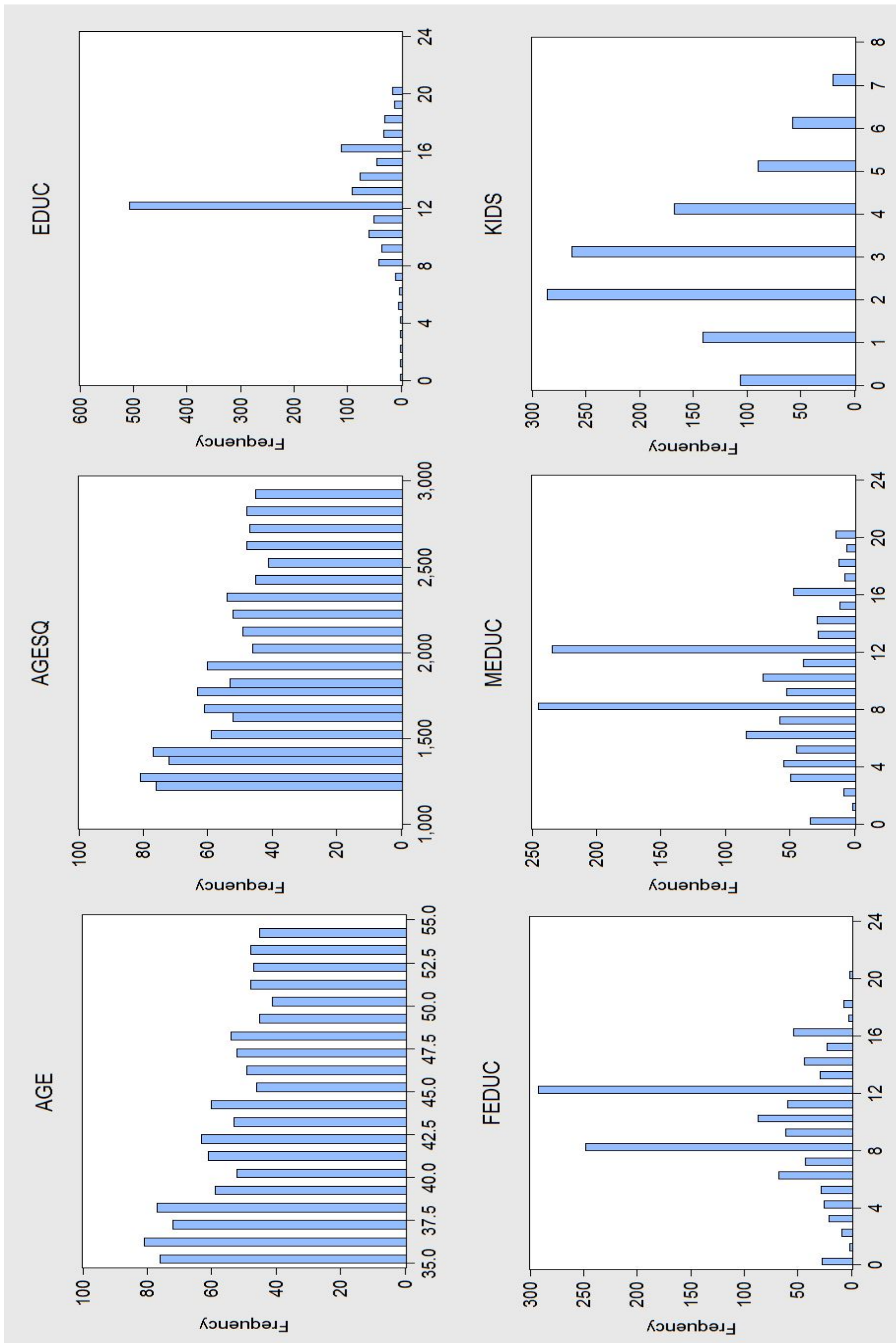


Figure 1: Descriptive statistics.

Sample: 1 1129

	AGE	AGESQ	EDUC	FEDUC	KIDS	MEDUC
Mean	43.48450	1924.935	12.69088	9.715678	2.743136	9.131975
Median	43.00000	1849.000	12.00000	10.00000	3.000000	8.000000
Maximum	54.00000	2916.000	20.00000	20.00000	7.000000	20.00000
Minimum	35.00000	1225.000	0.000000	0.000000	0.000000	0.000000
Std. Dev.	5.836421	515.8564	2.640236	3.495150	1.653899	4.016956
Skewness	0.217548	0.369616	0.095915	-0.382531	0.356569	0.123342
Kurtosis	1.813318	1.913573	4.501157	3.313065	2.739184	3.072281
Jarque-Bera	75.15017	81.23097	107.7381	32.14497	27.12376	3.108396
Probability	0.000000	0.000000	0.000000	0.000000	0.000001	0.211359
Sum	49094.00	2173252.	14328.00	10969.00	3097.000	10310.00
Sum Sq. Dev.	38423.98	3.00E+08	7863.116	13779.73	3085.509	18201.34
Observations	1129	1129	1129	1129	1129	1129

Figure 2: Descriptive statistics.

Sample: 1 1129

	BLACK	EAST	FARM	NORTHCEN	OTHRURAL	SMCITY	TOWN	WEST
Mean	0.085031	0.248893	0.198406	0.319752	0.101860	0.125775	0.317095	0.108060
Median	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Maximum	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
Minimum	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Std. Dev.	0.279051	0.432563	0.398976	0.466587	0.302598	0.331743	0.465551	0.310594
Skewness	2.975458	1.161534	1.512512	0.772966	2.632642	2.257116	0.786106	2.524926
Kurtosis	9.853350	2.349162	3.287692	1.597476	7.930804	6.094575	1.617963	7.375250
Jarque-Bera	3875.377	273.7936	434.3609	204.9595	2447.862	1409.119	206.1309	2100.120
Probability	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Sum	96.00000	281.0000	224.0000	361.0000	115.0000	142.0000	358.0000	122.0000
Sum Sq. Dev.	87.83702	211.0611	179.5571	245.5695	103.2861	124.1399	244.4801	108.8167
Observations	1129	1129	1129	1129	1129	1129	1129	1129

Figure 3: Descriptive statistics.

Sample: 1 1129

	Y74	Y76	Y78	Y80	Y82	Y84
Mean	0.153233	0.134632	0.126661	0.125775	0.164748	0.156776
Median	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Maximum	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
Minimum	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Std. Dev.	0.360372	0.341482	0.332740	0.331743	0.371117	0.363750
Skewness	1.925350	2.140843	2.245025	2.257116	1.807522	1.887977
Kurtosis	4.706974	5.583210	6.040135	6.094575	4.267135	4.564455
Jarque-Bera	834.5970	1176.315	1383.164	1409.119	690.2976	785.8472
Probability	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Sum	173.0000	152.0000	143.0000	142.0000	186.0000	177.0000
Sum Sq. Dev.	146.4907	131.5359	124.8875	124.1399	155.3570	149.2507
Observations	1129	1129	1129	1129	1129	1129

Figure 4: Descriptive statistics.

Sample: 1 1129
Included observations: 1129
Tabulation Summary

Variable	Categories
EDUC	21
YEAR	7
Product of Categories	147

Count		YEAR							
		72	74	76	78	80	82	84	Total
	0	0	0	1	0	0	0	0	1
	1	0	0	0	1	0	0	0	1
	2	0	0	0	0	1	0	0	1
	3	0	0	1	0	0	0	0	1
	4	0	1	0	0	0	0	0	1
	5	0	1	1	0	2	0	0	4
	6	1	1	0	0	0	0	1	3
	7	1	3	1	1	0	3	2	11
	8	13	7	6	4	5	3	4	42
	9	6	2	7	2	7	6	5	35
EDUC	10	8	14	9	9	4	10	5	59
	11	10	6	9	9	3	8	5	50
	12	73	86	80	68	57	74	69	507
	13	8	15	4	10	20	14	20	91
	14	12	8	7	10	10	14	15	76
	15	6	7	3	7	4	8	10	45
	16	13	12	13	13	15	22	23	111
	17	2	6	3	3	4	9	5	32
	18	2	3	4	4	4	8	6	31
	19	1	0	2	1	3	2	3	12
	20	0	1	1	1	3	5	4	15
	Total	156	173	152	143	142	186	177	1129

Figure 5: Number of occurrences (counts) of each pair of EDUC (21 possible values, or "categories") and YEAR (7 possible values), e.g., 73 individuals were recorded with EDUC equal to 12, in the year 1972.

Dependent Variable: KIDS
Method: Least Squares
Date: 10/19/14 Time: 17:51
Sample: 1 1129
Included observations: 1129

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-7.742457	3.051767	-2.537040	0.0113
EDUC	-0.128427	0.018349	-6.999272	0.0000
AGE	0.532135	0.138386	3.845283	0.0001
AGESQ	-0.005804	0.001564	-3.710324	0.0002
BLACK	1.075658	0.173536	6.198484	0.0000
EAST	0.217324	0.132788	1.636626	0.1020
NORTHCEN	0.363114	0.120897	3.003501	0.0027
WEST	0.197603	0.166913	1.183867	0.2367
FARM	-0.052557	0.147190	-0.357072	0.7211
OTHRURAL	-0.162854	0.175442	-0.928248	0.3535
TOWN	0.084353	0.124531	0.677367	0.4983
SMCITY	0.211879	0.160296	1.321799	0.1865
Y74	0.268183	0.172716	1.552737	0.1208
Y76	-0.097379	0.179046	-0.543881	0.5866
Y78	-0.068666	0.181684	-0.377945	0.7055
Y80	-0.071305	0.182771	-0.390136	0.6965
Y82	-0.522484	0.172436	-3.030016	0.0025
Y84	-0.545166	0.174516	-3.123871	0.0018

R-squared	0.129512	Mean dependent var	2.743136
Adjusted R-squared	0.116192	S.D. dependent var	1.653899
S.E. of regression	1.554847	Akaike info criterion	3.736447
Sum squared resid	2685.898	Schwarz criterion	3.816627
Log likelihood	-2091.224	Hannan-Quinn criter.	3.766741
F-statistic	9.723282	Durbin-Watson stat	2.010694
Prob(F-statistic)	0.000000		

Figure 6: EQ01.

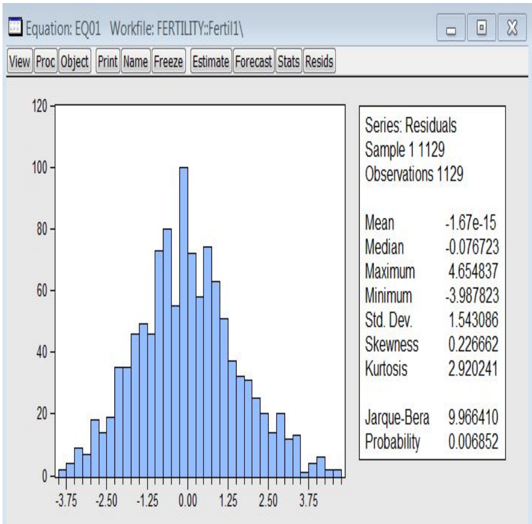
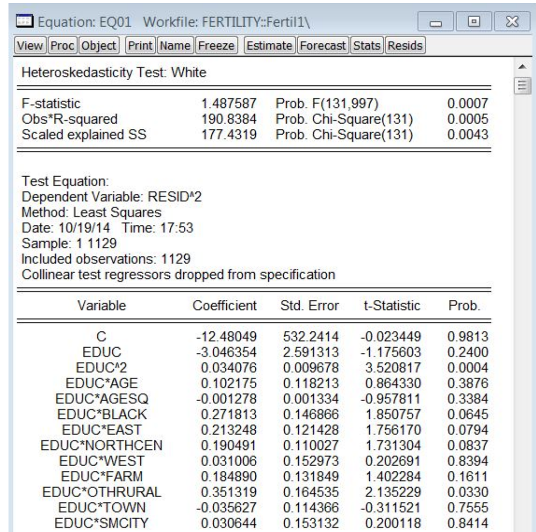


Figure 7: EQ01.


Equation: EQ01 Workfile: FERTILITY::Fertil1\

View Proc Object Print Name Freeze Estimate Forecast Stats Resids

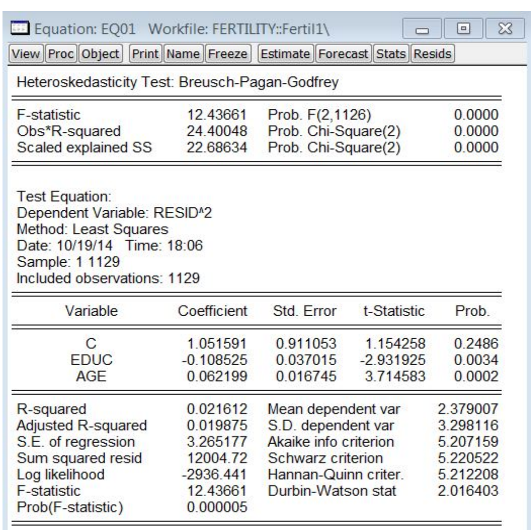
Heteroskedasticity Test: White

F-statistic	1.487587	Prob. F(131,997)	0.0007
Obs*R-squared	190.8384	Prob. Chi-Square(131)	0.0005
Scaled explained SS	177.4319	Prob. Chi-Square(131)	0.0043

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 10/19/14 Time: 17:53
Sample: 1 1129
Included observations: 1129
Collinear test regressors dropped from specification

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-12.48049	532.2414	-0.023449	0.9813
EDUC	-3.046354	2.591313	-1.175603	0.2400
EDUC^2	0.034076	0.009678	3.520817	0.0004
EDUC*AGE	0.102175	0.118213	0.864330	0.3876
EDUC*AGESQ	-0.001278	0.001334	-0.957811	0.3384
EDUC*BLACK	0.271813	0.146866	1.850757	0.0645
EDUC*EAST	0.213248	0.121428	1.756170	0.0794
EDUC*NORTHCEN	0.190491	0.110027	1.731304	0.0837
EDUC*WEST	0.031006	0.152973	0.202691	0.8394
EDUC*FARM	0.184890	0.131849	1.402284	0.1611
EDUC*OTHRURAL	0.351319	0.164535	2.135229	0.0330
EDUC*TOWN	-0.035627	0.114366	-0.311521	0.7555
EDUC*SMCITY	0.030644	0.153132	0.200118	0.8414

Figure 8: EQ01.


Equation: EQ01 Workfile: FERTILITY::Fertil1\

View Proc Object Print Name Freeze Estimate Forecast Stats Resids

Heteroskedasticity Test: Breusch-Pagan-Godfrey

F-statistic	12.43661	Prob. F(2,1126)	0.0000
Obs*R-squared	24.40048	Prob. Chi-Square(2)	0.0000
Scaled explained SS	22.68634	Prob. Chi-Square(2)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 10/19/14 Time: 18:06
Sample: 1 1129
Included observations: 1129

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.051591	0.911053	1.154258	0.2486
EDUC	-0.108525	0.037015	-2.931925	0.0034
AGE	0.062199	0.016745	3.714583	0.0002

R-squared	0.021612	Mean dependent var	2.379007
Adjusted R-squared	0.019875	S.D. dependent var	3.298116
S.E. of regression	3.265177	Akaike info criterion	5.207159
Sum squared resid	12004.72	Schwarz criterion	5.220522
Log likelihood	-2936.441	Hannan-Quinn criter.	5.212208
F-statistic	12.43661	Durbin-Watson stat	2.016403
Prob(F-statistic)	0.000005		

Figure 9: EQ01.

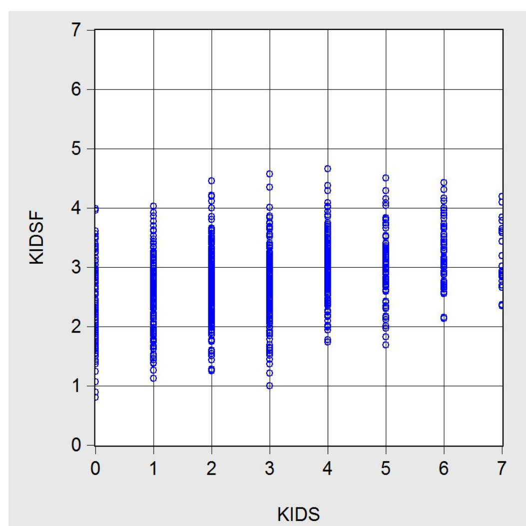


Figure 10: EQ01 (predicted against actual).

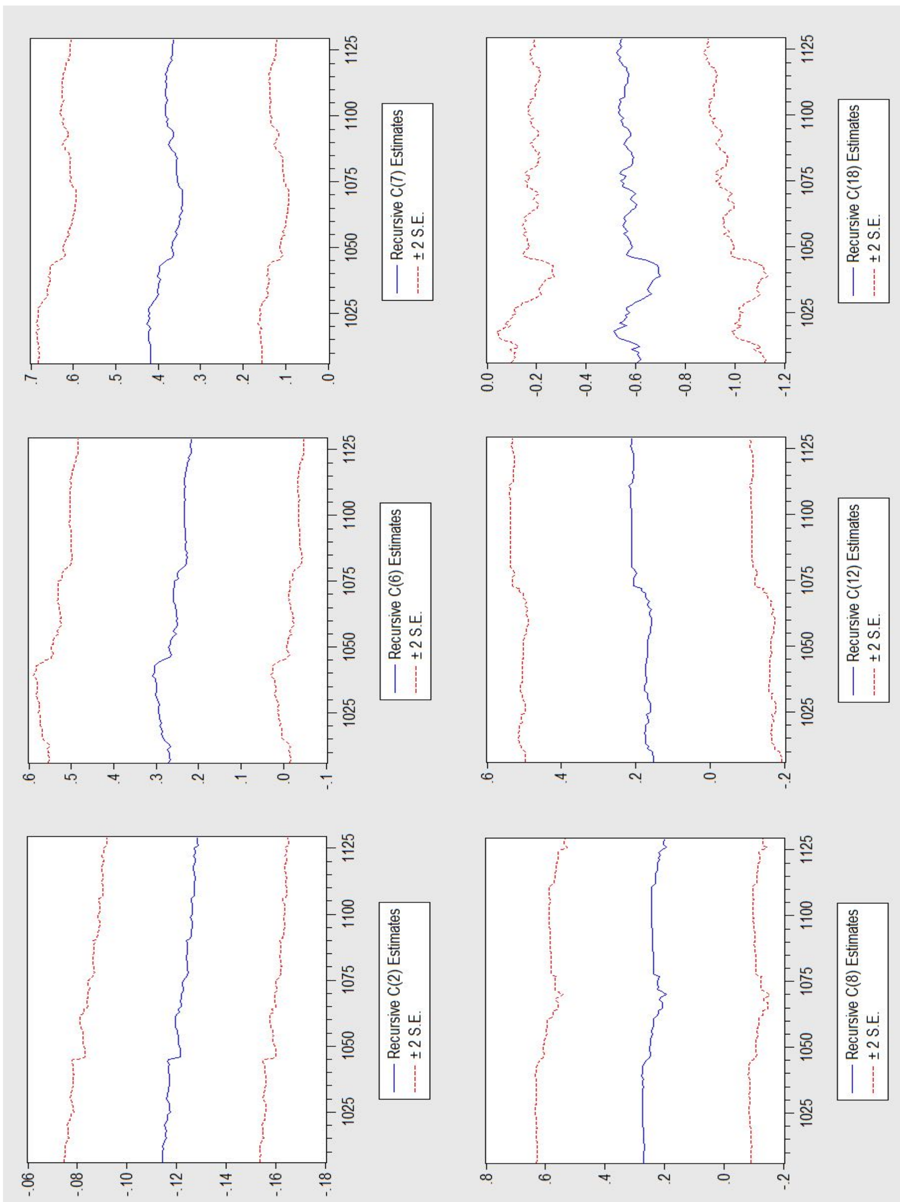


Figure 11: EQ01.

Dependent Variable: KIDS
Method: Least Squares
Date: 10/19/14 Time: 22:09
Sample: 1 1129
Included observations: 1129
White Heteroskedasticity-Consistent Standard Errors & Covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-7.742457	3.070656	-2.521434	0.0118
EDUC	-0.128427	0.021146	-6.073332	0.0000
AGE	0.532135	0.138937	3.830038	0.0001
AGESQ	-0.005804	0.001579	-3.675413	0.0002
BLACK	1.075658	0.201319	5.343055	0.0000
EAST	0.217324	0.127466	1.704956	0.0885
NORTHCEN	0.363114	0.116701	3.111482	0.0019
WEST	0.197603	0.162681	1.214665	0.2248
FARM	-0.052557	0.146084	-0.359776	0.7191
OTHRURAL	-0.162854	0.180855	-0.900468	0.3681
TOWN	0.084353	0.128476	0.656569	0.5116
SMCITY	0.211879	0.153964	1.376156	0.1691
Y74	0.268183	0.187512	1.430214	0.1529
Y76	-0.097379	0.199934	-0.487058	0.6263
Y78	-0.068666	0.197715	-0.347299	0.7284
Y80	-0.071305	0.193655	-0.368208	0.7128
Y82	-0.522484	0.187930	-2.780200	0.0055
Y84	-0.545166	0.185929	-2.932121	0.0034

R-squared	0.129512	Mean dependent var	2.743136
Adjusted R-squared	0.116192	S.D. dependent var	1.653899
S.E. of regression	1.554847	Akaike info criterion	3.736447
Sum squared resid	2685.898	Schwarz criterion	3.816627
Log likelihood	-2091.224	Hannan-Quinn criter.	3.766741
F-statistic	9.723282	Durbin-Watson stat	2.010694
Prob(F-statistic)	0.000000		

Figure 12: EQ02.

Dependent Variable: KIDS
Method: Least Squares
Date: 10/19/14 Time: 22:12
Sample: 1 1129
Included observations: 1129
White Heteroskedasticity-Consistent Standard Errors & Covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-8.487543	3.087589	-2.748923	0.0061
EDUC	-0.142879	0.021173	-6.748054	0.0000
AGE	0.562422	0.139981	4.017840	0.0001
AGESQ	-0.006092	0.001589	-3.834038	0.0001
BLACK	0.977559	0.202350	4.831040	0.0000
EAST	0.236293	0.129852	1.819705	0.0691
NORTHCEN	0.384749	0.117361	3.278325	0.0011
WEST	0.244703	0.166037	1.473787	0.1408
FARM	-0.054186	0.147759	-0.366719	0.7139
OTHRURAL	-0.167075	0.183860	-0.908710	0.3637
TOWN	0.084237	0.128800	0.654013	0.5132
SMCITY	0.183077	0.155546	1.176995	0.2394

R-squared	0.101919	Mean dependent var	2.743136
Adjusted R-squared	0.093075	S.D. dependent var	1.653899
S.E. of regression	1.575051	Akaike info criterion	3.757024
Sum squared resid	2771.037	Schwarz criterion	3.810478
Log likelihood	-2108.840	Hannan-Quinn criter.	3.777220
F-statistic	11.52391	Durbin-Watson stat	1.953542
Prob(F-statistic)	0.000000		

Figure 13: EQ03.

Dependent Variable: KIDS
Method: Least Squares
Date: 10/19/14 Time: 22:23
Sample: 1 1129
Included observations: 1129
White Heteroskedasticity-Consistent Standard Errors & Covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-8.477302	3.193861	-2.654249	0.0081
EDUC	-0.022515	0.066141	-0.340413	0.7336
AGE	0.507466	0.140034	3.623872	0.0003
AGESQ	-0.005525	0.001592	-3.469589	0.0005
BLACK	1.074055	0.200681	5.352044	0.0000
EAST	0.206056	0.127417	1.617182	0.1061
NORTHCEN	0.348287	0.116561	2.988015	0.0029
WEST	0.177122	0.163542	1.083037	0.2790
FARM	-0.072162	0.145270	-0.496747	0.6195
OTHRURAL	-0.191154	0.178438	-1.071260	0.2843
TOWN	0.088229	0.128574	0.686218	0.4927
SMCITY	0.205358	0.154396	1.330075	0.1838
Y74	0.946915	1.038280	0.912003	0.3620
Y76	1.019963	1.127292	0.904790	0.3658
Y78	1.805985	1.332366	1.355472	0.1755
Y80	1.114183	1.050826	1.060293	0.2892
Y82	1.199807	1.009239	1.188824	0.2348
Y84	1.671261	1.026677	1.627834	0.1038
Y74EDUC	-0.056425	0.081940	-0.688608	0.4912
Y76EDUC	-0.092100	0.089756	-1.026115	0.3051
Y78EDUC	-0.152387	0.103474	-1.472715	0.1411
Y80EDUC	-0.097905	0.083610	-1.170976	0.2419
Y82EDUC	-0.138945	0.079251	-1.753216	0.0798
Y84EDUC	-0.176097	0.079619	-2.211741	0.0272

R-squared	0.136468	Mean dependent var	2.743136
Adjusted R-squared	0.118494	S.D. dependent var	1.653899
S.E. of regression	1.552821	Akaike info criterion	3.739052
Sum squared resid	2664.435	Schwarz criterion	3.845959
Log likelihood	-2086.695	Hannan-Quinn criter.	3.779444
F-statistic	7.592560	Durbin-Watson stat	2.012728
Prob(F-statistic)	0.000000		

Figure 14: EQ04.

Wald Test:
Equation: EQ04

Test Statistic	Value	df	Probability
F-statistic	1.173090	(6, 1105)	0.3182
Chi-square	7.038541	6	0.3173

Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(19)	-0.056425	0.081940
C(20)	-0.092100	0.089756
C(21)	-0.152387	0.103474
C(22)	-0.097905	0.083610
C(23)	-0.138945	0.079251
C(24)	-0.176097	0.079619

Restrictions are linear in coefficients.

Figure 15: EQ04.

Areas Under the Normal Curve

Z	Cum p	Tail p	Z	Cum p	Tail p	Z	Cum p	Tail p	Z	Cum p	Tail p	Z	Cum p	Tail p
0.00	0.5000	0.5000	0.40	0.6554	0.3446	0.80	0.7881	0.2119	1.20	0.8849	0.1151	1.60	0.9452	0.0548
0.01	0.5040	0.4960	0.41	0.6591	0.3409	0.81	0.7910	0.2090	1.21	0.8869	0.1131	1.61	0.9463	0.0537
0.02	0.5080	0.4920	0.42	0.6628	0.3372	0.82	0.7939	0.2061	1.22	0.8888	0.1112	1.62	0.9474	0.0526
0.03	0.5120	0.4880	0.43	0.6664	0.3336	0.83	0.7967	0.2033	1.23	0.8907	0.1093	1.63	0.9484	0.0516
0.04	0.5160	0.4840	0.44	0.6700	0.3300	0.84	0.7995	0.2005	1.24	0.8925	0.1075	1.64	0.9495	0.0505
0.05	0.5199	0.4801	0.45	0.6736	0.3264	0.85	0.8023	0.1977	1.25	0.8944	0.1056	**1.65**	**0.9505**	**0.0495**
0.06	0.5239	0.4761	0.46	0.6772	0.3228	0.86	0.8051	0.1949	1.26	0.8962	0.1038	1.66	0.9515	0.0485
0.07	0.5279	0.4721	0.47	0.6808	0.3192	0.87	0.8078	0.1922	1.27	0.8980	0.1020	1.67	0.9525	0.0475
0.08	0.5319	0.4681	0.48	0.6844	0.3156	0.88	0.8106	0.1894	1.28	0.8997	0.1003	1.68	0.9535	0.0465
0.09	0.5359	0.4641	0.49	0.6879	0.3121	0.89	0.8133	0.1867	1.29	0.9015	0.0985	1.69	0.9545	0.0455
0.10	0.5398	0.4602	0.50	0.6915	0.3085	0.90	0.8159	0.1841	1.30	0.9032	0.0968	1.70	0.9554	0.0446
0.11	0.5438	0.4562	0.51	0.6950	0.3050	0.91	0.8186	0.1814	1.31	0.9049	0.0951	1.71	0.9564	0.0436
0.12	0.5478	0.4522	0.52	0.6985	0.3015	0.92	0.8212	0.1788	1.32	0.9066	0.0934	1.72	0.9573	0.0427
0.13	0.5517	0.4483	0.53	0.7019	0.2981	0.93	0.8238	0.1762	1.33	0.9082	0.0918	1.73	0.9582	0.0418
0.14	0.5557	0.4443	0.54	0.7054	0.2946	0.94	0.8264	0.1736	1.34	0.9099	0.0901	1.74	0.9591	0.0409
0.15	0.5596	0.4404	0.55	0.7088	0.2912	0.95	0.8289	0.1711	1.35	0.9115	0.0885	1.75	0.9599	0.0401
0.16	0.5636	0.4364	0.56	0.7123	0.2877	0.96	0.8315	0.1685	1.36	0.9131	0.0869	1.76	0.9608	0.0392
0.17	0.5675	0.4325	0.57	0.7157	0.2843	0.97	0.8340	0.1660	1.37	0.9147	0.0853	1.77	0.9616	0.0384
0.18	0.5714	0.4286	0.58	0.7190	0.2810	0.98	0.8365	0.1635	1.38	0.9162	0.0838	1.78	0.9625	0.0375
0.19	0.5753	0.4247	0.59	0.7224	0.2776	0.99	0.8389	0.1611	1.39	0.9177	0.0823	1.79	0.9633	0.0367
0.20	0.5793	0.4207	0.60	0.7257	0.2743	1.00	0.8413	0.1587	1.40	0.9192	0.0808	1.80	0.9641	0.0359
0.21	0.5832	0.4168	0.61	0.7291	0.2709	1.01	0.8438	0.1562	1.41	0.9207	0.0793	1.81	0.9649	0.0351
0.22	0.5871	0.4129	0.62	0.7324	0.2676	1.02	0.8461	0.1539	1.42	0.9222	0.0778	1.82	0.9656	0.0344
0.23	0.5910	0.4090	0.63	0.7357	0.2643	1.03	0.8485	0.1515	1.43	0.9236	0.0764	1.83	0.9664	0.0336
0.24	0.5948	0.4052	0.64	0.7389	0.2611	1.04	0.8508	0.1492	1.44	0.9251	0.0749	1.84	0.9671	0.0329
0.25	0.5987	0.4013	0.65	0.7422	0.2578	1.05	0.8531	0.1469	1.45	0.9265	0.0735	1.85	0.9678	0.0322
0.26	0.6026	0.3974	0.66	0.7454	0.2546	1.06	0.8554	0.1446	1.46	0.9279	0.0721	1.86	0.9686	0.0314
0.27	0.6064	0.3936	0.67	0.7486	0.2514	1.07	0.8577	0.1423	1.47	0.9292	0.0708	1.87	0.9693	0.0307
0.28	0.6103	0.3897	0.68	0.7517	0.2483	1.08	0.8599	0.1401	1.48	0.9306	0.0694	1.88	0.9699	0.0301
0.29	0.6141	0.3859	0.69	0.7549	0.2451	1.09	0.8621	0.1379	1.49	0.9319	0.0681	1.89	0.9706	0.0294
0.30	0.6179	0.3821	0.70	0.7580	0.2420	1.10	0.8643	0.1357	1.50	0.9332	0.0668	1.90	0.9713	0.0287
0.31	0.6217	0.3783	0.71	0.7611	0.2389	1.11	0.8665	0.1335	1.51	0.9345	0.0655	1.91	0.9719	0.0281
0.32	0.6255	0.3745	0.72	0.7642	0.2358	1.12	0.8686	0.1314	1.52	0.9357	0.0643	1.92	0.9726	0.0274
0.33	0.6293	0.3707	0.73	0.7673	0.2327	1.13	0.8708	0.1292	1.53	0.9370	0.0630	1.93	0.9732	0.0268
0.34	0.6331	0.3669	0.74	0.7704	0.2296	1.14	0.8729	0.1271	1.54	0.9382	0.0618	1.94	0.9738	0.0262
0.35	0.6368	0.3632	0.75	0.7734	0.2266	1.15	0.8749	0.1251	1.55	0.9394	0.0606	1.95	0.9744	0.0256
0.36	0.6406	0.3594	0.76	0.7764	0.2236	1.16	0.8770	0.1230	1.56	0.9406	0.0594	**1.96**	**0.9750**	**0.0250**
0.37	0.6443	0.3557	0.77	0.7794	0.2206	1.17	0.8790	0.1210	1.57	0.9418	0.0582	1.97	0.9756	0.0244
0.38	0.6480	0.3520	0.78	0.7823	0.2177	1.18	0.8810	0.1190	1.58	0.9429	0.0571	1.98	0.9761	0.0239
0.39	0.6517	0.3483	0.79	0.7852	0.2148	1.19	0.8830	0.1170	1.59	0.9441	0.0559	1.99	0.9767	0.0233

Figure 16: Statistical table for N(0, 1). These tables are taken from `http://fsweb.berry.edu/academic/education/vbissonnette/tables/tables.html`

Critical Values of the t Distribution

df	2-tailed testing			1-tailed testing		
	• •			• •		
	0.1	0.05	0.01	0.1	0.05	0.01
5	2.015	2.571	4.032	1.476	2.015	3.365
6	1.943	2.447	3.707	1.440	1.943	3.143
7	1.895	2.365	3.499	1.415	1.895	2.998
8	1.860	2.306	3.355	1.397	1.860	2.896
9	1.833	2.262	3.250	1.383	1.833	2.821
10	1.812	2.228	3.169	1.372	1.812	2.764
11	1.796	2.201	3.106	1.363	1.796	2.718
12	1.782	2.179	3.055	1.356	1.782	2.681
13	1.771	2.160	3.012	1.350	1.771	2.650
14	1.761	2.145	2.977	1.345	1.761	2.624
15	1.753	2.131	2.947	1.341	1.753	2.602
16	1.746	2.120	2.921	1.337	1.746	2.583
17	1.740	2.110	2.898	1.333	1.740	2.567
18	1.734	2.101	2.878	1.330	1.734	2.552
19	1.729	2.093	2.861	1.328	1.729	2.539
20	1.725	2.086	2.845	1.325	1.725	2.528
21	1.721	2.080	2.831	1.323	1.721	2.518
22	1.717	2.074	2.819	1.321	1.717	2.508
23	1.714	2.069	2.807	1.319	1.714	2.500
24	1.711	2.064	2.797	1.318	1.711	2.492
25	1.708	2.060	2.787	1.316	1.708	2.485
26	1.706	2.056	2.779	1.315	1.706	2.479
27	1.703	2.052	2.771	1.314	1.703	2.473
28	1.701	2.048	2.763	1.313	1.701	2.467
29	1.699	2.045	2.756	1.311	1.699	2.462
30	1.697	2.042	2.750	1.310	1.697	2.457
40	1.684	2.021	2.704	1.303	1.684	2.423
50	1.676	2.009	2.678	1.299	1.676	2.403
60	1.671	2.000	2.660	1.296	1.671	2.390
80	1.664	1.990	2.639	1.292	1.664	2.374
100	1.660	1.984	2.626	1.290	1.660	2.364
120	1.658	1.980	2.617	1.289	1.658	2.358
• •	1.645	1.960	2.576	1.282	1.645	2.327

Figure 17: Statistical table for Student's $t(r)$.

Critical Values of the F Distribution
(α = .05)

df within	df between									
	1	2	3	4	5	6	7	8	12	24	∞
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.68	4.53	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.00	3.84	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.57	3.41	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.28	3.12	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.07	2.90	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	2.91	2.74	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.79	2.61	2.41
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.69	2.51	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.60	2.42	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.53	2.35	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.48	2.29	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.42	2.24	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.38	2.19	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.34	2.15	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.31	2.11	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.28	2.08	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.25	2.05	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.23	2.03	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.20	2.01	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.18	1.98	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.16	1.96	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.15	1.95	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.13	1.93	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.12	1.91	1.66
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.10	1.90	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.09	1.89	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.00	1.79	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	1.92	1.70	1.39
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	1.88	1.65	1.33
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.85	1.63	1.28
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.83	1.61	1.26
∞	3.84	3.00	2.61	2.37	2.22	2.10	2.01	1.94	1.75	1.52	1.00

Figure 18: Statistical table for $F(m,p)$ at the 5% level.

Critical Values of the F Distribution
(α = .01)

df within	df between										
	1	2	3	4	5	6	7	8	12	24	∞
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	9.89	9.47	9.02
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.72	7.31	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.47	6.07	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.67	5.28	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.11	4.73	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.71	4.33	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.40	4.02	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.16	3.78	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	3.96	3.59	3.17
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	3.80	3.43	3.01
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.67	3.29	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.55	3.18	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.46	3.08	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.37	3.00	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.30	2.92	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.23	2.86	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.17	2.80	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.12	2.75	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.07	2.70	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.03	2.66	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	2.99	2.62	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	2.96	2.58	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	2.93	2.55	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	2.90	2.52	2.07
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	2.87	2.49	2.04
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	2.84	2.47	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.66	2.29	1.81
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.50	2.12	1.60
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.42	2.03	1.50
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.37	1.98	1.43
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.34	1.95	1.38
∞	6.64	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.19	1.79	1.00

Figure 19: Statistical table for $F(m, p)$ at the 1% level.

Critical Values of the χ^2 Distribution

df	Area in the Upper Tail					
	0.99	0.95	0.9	0.1	**0.05**	**0.01**
1	0.000	0.004	0.016	2.706	3.841	6.635
2	0.020	0.103	0.211	4.605	5.991	9.210
3	0.115	0.352	0.584	6.251	7.815	11.345
4	0.297	0.711	1.064	7.779	9.488	13.277
5	0.554	1.145	1.610	9.236	11.070	15.086
6	0.872	1.635	2.204	10.645	12.592	16.812
7	1.239	2.167	2.833	12.017	14.067	18.475
8	1.646	2.733	3.490	13.362	15.507	20.090
9	2.088	3.325	4.168	14.684	16.919	21.666
10	2.558	3.940	4.865	15.987	18.307	23.209
11	3.053	4.575	5.578	17.275	19.675	24.725
12	3.571	5.226	6.304	18.549	21.026	26.217
13	4.107	5.892	7.042	19.812	22.362	27.688
14	4.660	6.571	7.790	21.064	23.685	29.141
15	5.229	7.261	8.547	22.307	24.996	30.578
16	5.812	7.962	9.312	23.542	26.296	32.000
17	6.408	8.672	10.085	24.769	27.587	33.409
18	7.015	9.390	10.865	25.989	28.869	34.805
19	7.633	10.117	11.651	27.204	30.144	36.191
20	8.260	10.851	12.443	28.412	31.410	37.566
21	8.897	11.591	13.240	29.615	32.671	38.932
22	9.542	12.338	14.041	30.813	33.924	40.289
23	10.196	13.091	14.848	32.007	35.172	41.638
24	10.856	13.848	15.659	33.196	36.415	42.980
25	11.524	14.611	16.473	34.382	37.652	44.314

Figure 20: Statistical table for $\chi^2(q)$.