**Topic: Heteroscedasticity**

- This problem set deals with the detection of heteroscedasticity in cross-sectional data, both visually and by use of several statistical diagnostic tests.
- We use data on monthly credit card expenditure, for $n = 100$ individuals, available as `credit_card.txt` on the website.
- The variables are: Y1 (number of derogatory/negative reports), Y2 (indicator variable: credit card application accepted? 1 = yes, 0 = no), X1 (age in years), X2 ($0.0001 \times$ income, in scaled U.S. dollars), X3 (average monthly credit card expenditure, in U.S. dollars), X4 (indicator variable: individuals owns / rents home? 1 = owns, 0 = rents), X5 (indicator variable: individual self-employed? 1 = yes, 0 = no).
- Refer to figures 1 – 4, and perform the following:[1]
  1. Perform a careful descriptive analysis of the dataset. In particular, (a) what features of interest can you find for each of the variables?, (b) approximately how many individuals have never had a credit card?! - look for evidence of first-time applications, (c) consider bivariate scatterplots and correlations of Y2 against each of the other variables - interpret the signs of the correlations, and (d) run a regression of Y2 on a constant, and all of the other variables - interpret the signs and magnitudes of the estimated coefficients, examine the significance of the variables, and compare your results with part (c) above.

[1] Note that not all of the necessary steps are shown in the figures!

2. Run a linear regression of monthly expenditure on a constant, age, scaled income, scaled income squared, and the home ownership indicator (eq01). Plot the estimated residuals $\widehat{u}_i$ (resid_eq01) against scaled income, with kernel densities superimposed on the axes (graph01), and interpret the results. Test **manually** for normality of the estimated residuals, and compare your result with the EViews 6 (menu) version of the Jarque-Bera test. What do you notice?!

3. Perform White's $nR^2$ general test for heteroscedasticity **manually**, at the 95% level, where $R^2$ is computed from the regression of the squared fitted residuals $\widehat{u}_i^2$ on a constant, all explanatory variables, and all squares and cross-products of explanatory variables (eq02): explain why $\text{X4}^2$ is not included.

   White's $nR^2$ test is for the null $H_0 : \sigma_i^2 = \sigma^2$ for all $i = 1, 2, \ldots, n$, against the alternative $H_1 :$ not $H_0$. Interpret the results. Check your solution against the EViews 6 (menu) version of this test.

4. What is the estimated sum of squared residuals ($\widehat{u}'\widehat{u}$) from eq01? An alternative test for heteroscedasticity is due to Breusch and Pagan, and Godfrey: it is a Lagrange multiplier test of $\alpha = 0$ (homoscedasticity) in $H_0 : \sigma_i^2 = \sigma^2 f(\alpha_0 + \alpha' z_i)$, against $H_1 :$ not $H_0$, where $z_i$ is some vector of variables excluding a constant. The test statistic is:

$$\text{BPG}_{\texttt{LM}} = \frac{1}{2}\texttt{ESS} = \frac{1}{2}(\widehat{y}'\widehat{y} - n\bar{y}^2) \sim \chi^2(m),$$

   where `ESS` is the explained sum of squares from the regression of $y_i := n\widehat{u}_i^2/\widehat{u}'\widehat{u}$ on a constant and $z_i$, and $m$ is the number of variables in (= dimension of) $z_i$. Use $z_i = (\text{X2}_i, \text{X2}_i^2)'$, and perform the test **manually** at the 95% level. Explain carefully what you notice about the mean of $y$. Interpret your results. Check your results against the EViews 6 (menu) version of the test. What do you notice?! (hint 1: compare the EViews 6 auxiliary regression for the $BPG_{LM}$ test against your manual version - could rounding error be a problem here?; hint 2: look at the EViews 6 help page for the $BPG_{LM}$ test - is this what EViews 6 actually does?!; hint 3: using your answer(s) to hint 2, try to calculate the EViews 6 'scaled explained SS' test result manually).[2]

[2]These problems do not affect the outcome of the test here, although this will not generally be true.

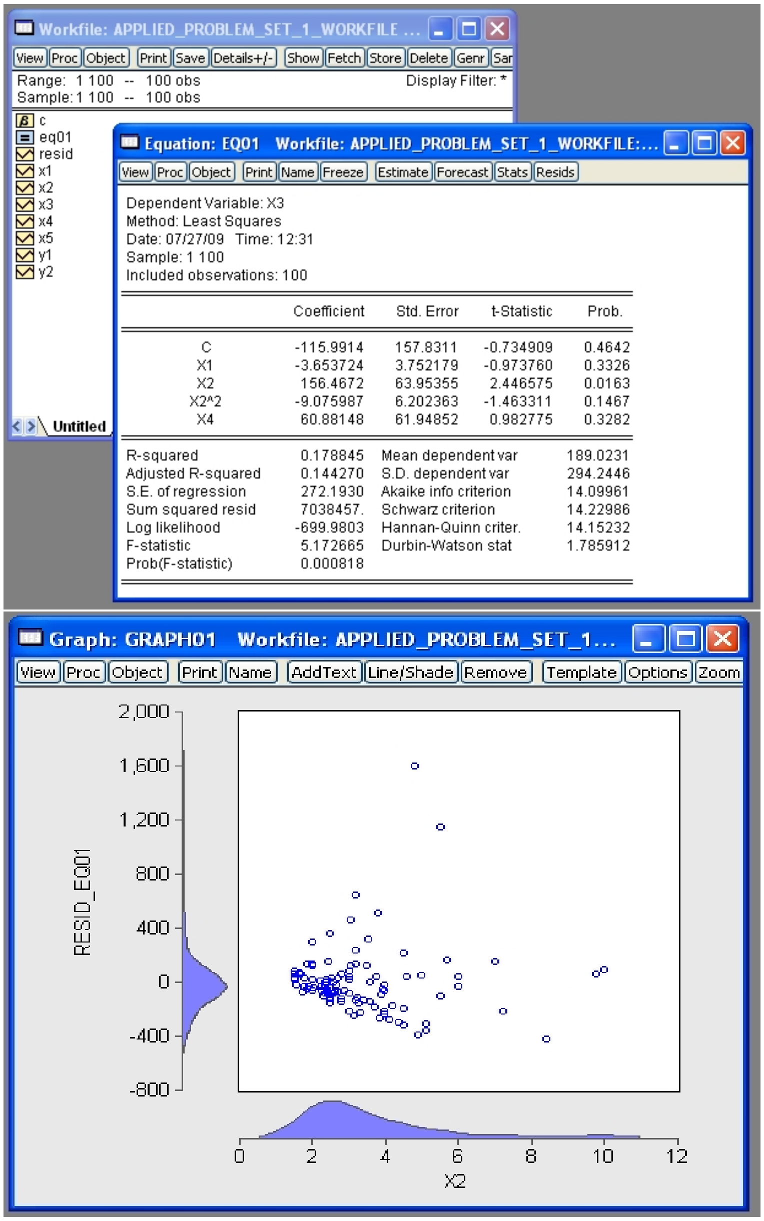


Figure 1: Run a linear regression of X3 on a constant, X1, X2, X2 squared, and X4. Plot the fitted residuals against X2, with kernel densities superimposed on the axes.

Equation: EQ02 Workfile: APPLIED_PROBLEM_SET_1_...

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: RESID_EQ01^2
Method: Least Squares
Date: 07/27/09 Time: 12:57
Sample: 1 100
Included observations: 100

| | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| C | 876511.9 | 913863.8 | 0.959128 | 0.3402 |
| X1 | 28775.90 | 31660.00 | 0.908904 | 0.3659 |
| X2 | -1509045. | 778264.9 | -1.938986 | 0.0557 |
| X2^2 | 498964.2 | 253154.3 | 1.970989 | 0.0519 |
| X4 | 195763.1 | 474111.1 | 0.412905 | 0.6807 |
| X1^2 | -644.2271 | 425.9743 | -1.512361 | 0.1341 |
| X2^4 | 2820.726 | 1630.189 | 1.730306 | 0.0871 |
| X1*X2 | 6853.915 | 11227.53 | 0.610456 | 0.5432 |
| X1*(X2^2) | -647.8628 | 1274.148 | -0.508467 | 0.6124 |
| X1*X4 | 5681.491 | 8776.134 | 0.647380 | 0.5191 |
| X2^3 | -63934.08 | 34454.00 | -1.855636 | 0.0669 |
| X2*X4 | -177650.5 | 199416.6 | -0.890851 | 0.3755 |
| (X2^2)*X4 | 11325.35 | 21530.66 | 0.526010 | 0.6002 |

| | | | |
|---|---|---|---|
| R-squared | 0.146539 | Mean dependent var | 70384.57 |
| Adjusted R-squared | 0.028820 | S.D. dependent var | 287729.4 |
| S.E. of regression | 283552.9 | Akaike info criterion | 28.06892 |
| Sum squared resid | 6.99E+12 | Schwarz criterion | 28.40760 |
| Log likelihood | -1390.446 | Hannan-Quinn criter. | 28.20599 |
| F-statistic | 1.244819 | Durbin-Watson stat | 1.745177 |
| Prob(F-statistic) | 0.266541 | | |

Figure 2: Auxiliary regression for White's $nR^2$ general test for heteroscedasticity.

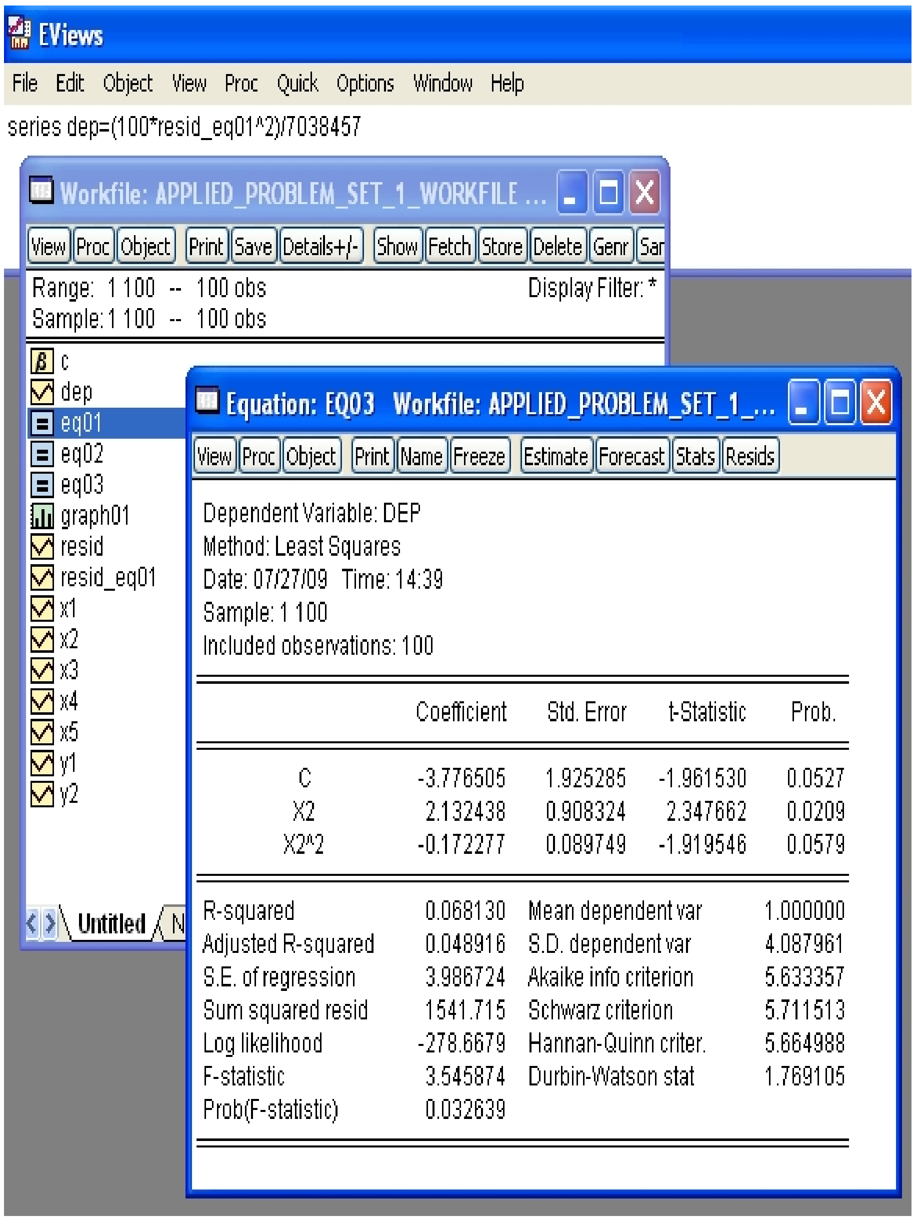


Figure 3: Computations required for Breusch-Pagan / Godfrey heteroscedasticity test (1).

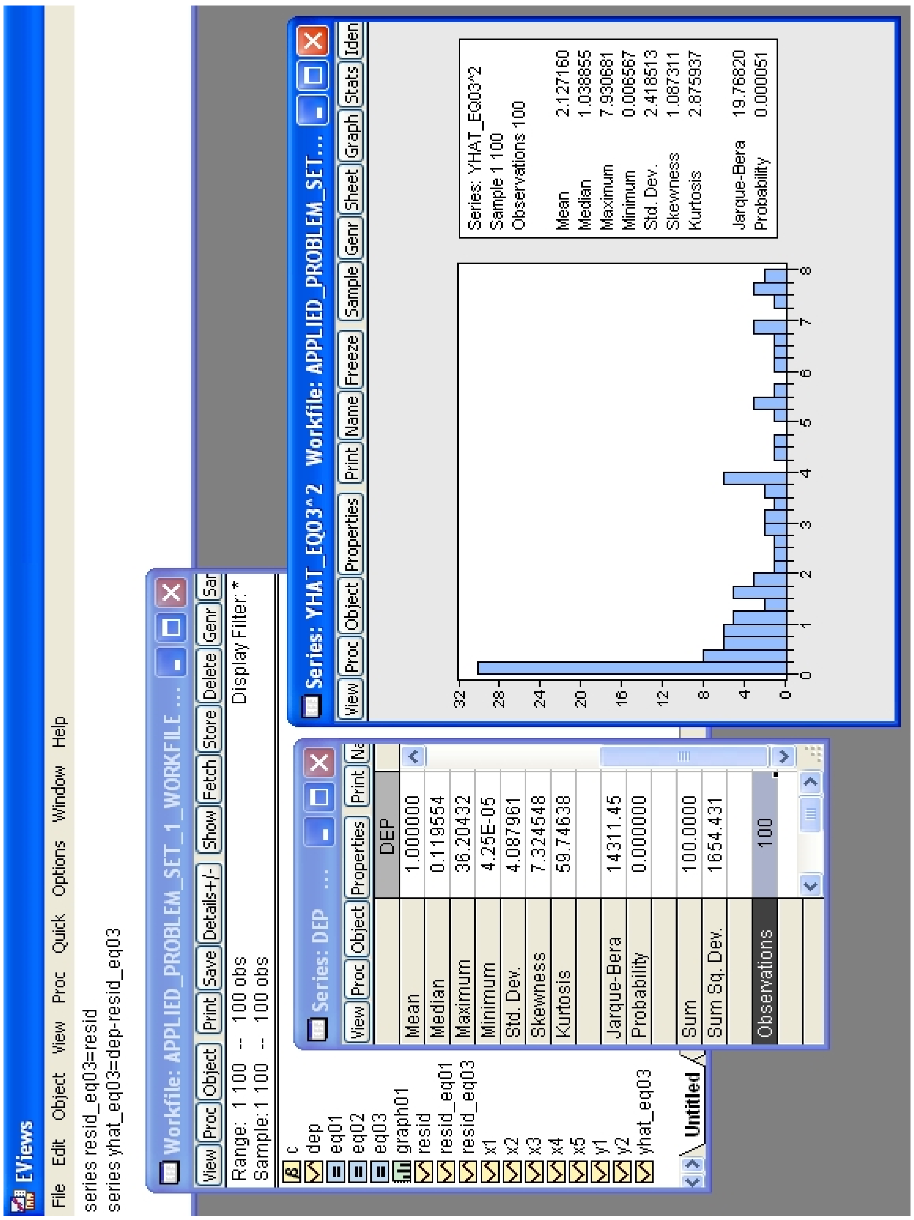


Figure 4: Computations required for Breusch-Pagan / Godfrey heteroscedasticity test (2).

**Areas Under the Normal Curve**

| Z | Cum p | Tail p | Z | Cum p | Tail p | Z | Cum p | Tail p | Z | Cum p | Tail p | Z | Cum p | Tail p |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 0.5000 | 0.5000 | 0.40 | 0.6554 | 0.3446 | 0.80 | 0.7881 | 0.2119 | 1.20 | 0.8849 | 0.1151 | 1.60 | 0.9452 | 0.0548 |
| 0.01 | 0.5040 | 0.4960 | 0.41 | 0.6591 | 0.3409 | 0.81 | 0.7910 | 0.2090 | 1.21 | 0.8869 | 0.1131 | 1.61 | 0.9463 | 0.0537 |
| 0.02 | 0.5080 | 0.4920 | 0.42 | 0.6628 | 0.3372 | 0.82 | 0.7939 | 0.2061 | 1.22 | 0.8888 | 0.1112 | 1.62 | 0.9474 | 0.0526 |
| 0.03 | 0.5120 | 0.4880 | 0.43 | 0.6664 | 0.3336 | 0.83 | 0.7967 | 0.2033 | 1.23 | 0.8907 | 0.1093 | 1.63 | 0.9484 | 0.0516 |
| 0.04 | 0.5160 | 0.4840 | 0.44 | 0.6700 | 0.3300 | 0.84 | 0.7995 | 0.2005 | 1.24 | 0.8925 | 0.1075 | 1.64 | 0.9495 | 0.0505 |
| 0.05 | 0.5199 | 0.4801 | 0.45 | 0.6736 | 0.3264 | 0.85 | 0.8023 | 0.1977 | 1.25 | 0.8944 | 0.1056 | **1.65** | **0.9505** | **0.0495** |
| 0.06 | 0.5239 | 0.4761 | 0.46 | 0.6772 | 0.3228 | 0.86 | 0.8051 | 0.1949 | 1.26 | 0.8962 | 0.1038 | 1.66 | 0.9515 | 0.0485 |
| 0.07 | 0.5279 | 0.4721 | 0.47 | 0.6808 | 0.3192 | 0.87 | 0.8078 | 0.1922 | 1.27 | 0.8980 | 0.1020 | 1.67 | 0.9525 | 0.0475 |
| 0.08 | 0.5319 | 0.4681 | 0.48 | 0.6844 | 0.3156 | 0.88 | 0.8106 | 0.1894 | 1.28 | 0.8997 | 0.1003 | 1.68 | 0.9535 | 0.0465 |
| 0.09 | 0.5359 | 0.4641 | 0.49 | 0.6879 | 0.3121 | 0.89 | 0.8133 | 0.1867 | 1.29 | 0.9015 | 0.0985 | 1.69 | 0.9545 | 0.0455 |
| 0.10 | 0.5398 | 0.4602 | 0.50 | 0.6915 | 0.3085 | 0.90 | 0.8159 | 0.1841 | 1.30 | 0.9032 | 0.0968 | 1.70 | 0.9554 | 0.0446 |
| 0.11 | 0.5438 | 0.4562 | 0.51 | 0.6950 | 0.3050 | 0.91 | 0.8186 | 0.1814 | 1.31 | 0.9049 | 0.0951 | 1.71 | 0.9564 | 0.0436 |
| 0.12 | 0.5478 | 0.4522 | 0.52 | 0.6985 | 0.3015 | 0.92 | 0.8212 | 0.1788 | 1.32 | 0.9066 | 0.0934 | 1.72 | 0.9573 | 0.0427 |
| 0.13 | 0.5517 | 0.4483 | 0.53 | 0.7019 | 0.2981 | 0.93 | 0.8238 | 0.1762 | 1.33 | 0.9082 | 0.0918 | 1.73 | 0.9582 | 0.0418 |
| 0.14 | 0.5557 | 0.4443 | 0.54 | 0.7054 | 0.2946 | 0.94 | 0.8264 | 0.1736 | 1.34 | 0.9099 | 0.0901 | 1.74 | 0.9591 | 0.0409 |
| 0.15 | 0.5596 | 0.4404 | 0.55 | 0.7088 | 0.2912 | 0.95 | 0.8289 | 0.1711 | 1.35 | 0.9115 | 0.0885 | 1.75 | 0.9599 | 0.0401 |
| 0.16 | 0.5636 | 0.4364 | 0.56 | 0.7123 | 0.2877 | 0.96 | 0.8315 | 0.1685 | 1.36 | 0.9131 | 0.0869 | 1.76 | 0.9608 | 0.0392 |
| 0.17 | 0.5675 | 0.4325 | 0.57 | 0.7157 | 0.2843 | 0.97 | 0.8340 | 0.1660 | 1.37 | 0.9147 | 0.0853 | 1.77 | 0.9616 | 0.0384 |
| 0.18 | 0.5714 | 0.4286 | 0.58 | 0.7190 | 0.2810 | 0.98 | 0.8365 | 0.1635 | 1.38 | 0.9162 | 0.0838 | 1.78 | 0.9625 | 0.0375 |
| 0.19 | 0.5753 | 0.4247 | 0.59 | 0.7224 | 0.2776 | 0.99 | 0.8389 | 0.1611 | 1.39 | 0.9177 | 0.0823 | 1.79 | 0.9633 | 0.0367 |
| 0.20 | 0.5793 | 0.4207 | 0.60 | 0.7257 | 0.2743 | 1.00 | 0.8413 | 0.1587 | 1.40 | 0.9192 | 0.0808 | 1.80 | 0.9641 | 0.0359 |
| 0.21 | 0.5832 | 0.4168 | 0.61 | 0.7291 | 0.2709 | 1.01 | 0.8438 | 0.1562 | 1.41 | 0.9207 | 0.0793 | 1.81 | 0.9649 | 0.0351 |
| 0.22 | 0.5871 | 0.4129 | 0.62 | 0.7324 | 0.2676 | 1.02 | 0.8461 | 0.1539 | 1.42 | 0.9222 | 0.0778 | 1.82 | 0.9656 | 0.0344 |
| 0.23 | 0.5910 | 0.4090 | 0.63 | 0.7357 | 0.2643 | 1.03 | 0.8485 | 0.1515 | 1.43 | 0.9236 | 0.0764 | 1.83 | 0.9664 | 0.0336 |
| 0.24 | 0.5948 | 0.4052 | 0.64 | 0.7389 | 0.2611 | 1.04 | 0.8508 | 0.1492 | 1.44 | 0.9251 | 0.0749 | 1.84 | 0.9671 | 0.0329 |
| 0.25 | 0.5987 | 0.4013 | 0.65 | 0.7422 | 0.2578 | 1.05 | 0.8531 | 0.1469 | 1.45 | 0.9265 | 0.0735 | 1.85 | 0.9678 | 0.0322 |
| 0.26 | 0.6026 | 0.3974 | 0.66 | 0.7454 | 0.2546 | 1.06 | 0.8554 | 0.1446 | 1.46 | 0.9279 | 0.0721 | 1.86 | 0.9686 | 0.0314 |
| 0.27 | 0.6064 | 0.3936 | 0.67 | 0.7486 | 0.2514 | 1.07 | 0.8577 | 0.1423 | 1.47 | 0.9292 | 0.0708 | 1.87 | 0.9693 | 0.0307 |
| 0.28 | 0.6103 | 0.3897 | 0.68 | 0.7517 | 0.2483 | 1.08 | 0.8599 | 0.1401 | 1.48 | 0.9306 | 0.0694 | 1.88 | 0.9699 | 0.0301 |
| 0.29 | 0.6141 | 0.3859 | 0.69 | 0.7549 | 0.2451 | 1.09 | 0.8621 | 0.1379 | 1.49 | 0.9319 | 0.0681 | 1.89 | 0.9706 | 0.0294 |
| 0.30 | 0.6179 | 0.3821 | 0.70 | 0.7580 | 0.2420 | 1.10 | 0.8643 | 0.1357 | 1.50 | 0.9332 | 0.0668 | 1.90 | 0.9713 | 0.0287 |
| 0.31 | 0.6217 | 0.3783 | 0.71 | 0.7611 | 0.2389 | 1.11 | 0.8665 | 0.1335 | 1.51 | 0.9345 | 0.0655 | 1.91 | 0.9719 | 0.0281 |
| 0.32 | 0.6255 | 0.3745 | 0.72 | 0.7642 | 0.2358 | 1.12 | 0.8686 | 0.1314 | 1.52 | 0.9357 | 0.0643 | 1.92 | 0.9726 | 0.0274 |
| 0.33 | 0.6293 | 0.3707 | 0.73 | 0.7673 | 0.2327 | 1.13 | 0.8708 | 0.1292 | 1.53 | 0.9370 | 0.0630 | 1.93 | 0.9732 | 0.0268 |
| 0.34 | 0.6331 | 0.3669 | 0.74 | 0.7704 | 0.2296 | 1.14 | 0.8729 | 0.1271 | 1.54 | 0.9382 | 0.0618 | 1.94 | 0.9738 | 0.0262 |
| 0.35 | 0.6368 | 0.3632 | 0.75 | 0.7734 | 0.2266 | 1.15 | 0.8749 | 0.1251 | 1.55 | 0.9394 | 0.0606 | 1.95 | 0.9744 | 0.0256 |
| 0.36 | 0.6406 | 0.3594 | 0.76 | 0.7764 | 0.2236 | 1.16 | 0.8770 | 0.1230 | 1.56 | 0.9406 | 0.0594 | **1.96** | **0.9750** | **0.0250** |
| 0.37 | 0.6443 | 0.3557 | 0.77 | 0.7794 | 0.2206 | 1.17 | 0.8790 | 0.1210 | 1.57 | 0.9418 | 0.0582 | 1.97 | 0.9756 | 0.0244 |
| 0.38 | 0.6480 | 0.3520 | 0.78 | 0.7823 | 0.2177 | 1.18 | 0.8810 | 0.1190 | 1.58 | 0.9429 | 0.0571 | 1.98 | 0.9761 | 0.0239 |
| 0.39 | 0.6517 | 0.3483 | 0.79 | 0.7852 | 0.2148 | 1.19 | 0.8830 | 0.1170 | 1.59 | 0.9441 | 0.0559 | 1.99 | 0.9767 | 0.0233 |

Figure 5: Statistical table for N(0, 1). These tables have been taken from: `http://fsweb.berry.edu/academic/education/vbissonnette/tables/tables.html`.

**Critical Values of the t Distribution**

| df | 2-tailed testing | | | 1-tailed testing | | |
|---|---|---|---|---|---|---|
| | • • | | | • • | | |
| | 0.1 | 0.05 | 0.01 | 0.1 | 0.05 | 0.01 |
| 5 | 2.015 | 2.571 | 4.032 | 1.476 | 2.015 | 3.365 |
| 6 | 1.943 | 2.447 | 3.707 | 1.440 | 1.943 | 3.143 |
| 7 | 1.895 | 2.365 | 3.499 | 1.415 | 1.895 | 2.998 |
| 8 | 1.860 | 2.306 | 3.355 | 1.397 | 1.860 | 2.896 |
| 9 | 1.833 | 2.262 | 3.250 | 1.383 | 1.833 | 2.821 |
| 10 | 1.812 | 2.228 | 3.169 | 1.372 | 1.812 | 2.764 |
| 11 | 1.796 | 2.201 | 3.106 | 1.363 | 1.796 | 2.718 |
| 12 | 1.782 | 2.179 | 3.055 | 1.356 | 1.782 | 2.681 |
| 13 | 1.771 | 2.160 | 3.012 | 1.350 | 1.771 | 2.650 |
| 14 | 1.761 | 2.145 | 2.977 | 1.345 | 1.761 | 2.624 |
| 15 | 1.753 | 2.131 | 2.947 | 1.341 | 1.753 | 2.602 |
| 16 | 1.746 | 2.120 | 2.921 | 1.337 | 1.746 | 2.583 |
| 17 | 1.740 | 2.110 | 2.898 | 1.333 | 1.740 | 2.567 |
| 18 | 1.734 | 2.101 | 2.878 | 1.330 | 1.734 | 2.552 |
| 19 | 1.729 | 2.093 | 2.861 | 1.328 | 1.729 | 2.539 |
| 20 | 1.725 | 2.086 | 2.845 | 1.325 | 1.725 | 2.528 |
| 21 | 1.721 | 2.080 | 2.831 | 1.323 | 1.721 | 2.518 |
| 22 | 1.717 | 2.074 | 2.819 | 1.321 | 1.717 | 2.508 |
| 23 | 1.714 | 2.069 | 2.807 | 1.319 | 1.714 | 2.500 |
| 24 | 1.711 | 2.064 | 2.797 | 1.318 | 1.711 | 2.492 |
| 25 | 1.708 | 2.060 | 2.787 | 1.316 | 1.708 | 2.485 |
| 26 | 1.706 | 2.056 | 2.779 | 1.315 | 1.706 | 2.479 |
| 27 | 1.703 | 2.052 | 2.771 | 1.314 | 1.703 | 2.473 |
| 28 | 1.701 | 2.048 | 2.763 | 1.313 | 1.701 | 2.467 |
| 29 | 1.699 | 2.045 | 2.756 | 1.311 | 1.699 | 2.462 |
| 30 | 1.697 | 2.042 | 2.750 | 1.310 | 1.697 | 2.457 |
| 40 | 1.684 | 2.021 | 2.704 | 1.303 | 1.684 | 2.423 |
| 50 | 1.676 | 2.009 | 2.678 | 1.299 | 1.676 | 2.403 |
| 60 | 1.671 | 2.000 | 2.660 | 1.296 | 1.671 | 2.390 |
| 80 | 1.664 | 1.990 | 2.639 | 1.292 | 1.664 | 2.374 |
| 100 | 1.660 | 1.984 | 2.626 | 1.290 | 1.660 | 2.364 |
| 120 | 1.658 | 1.980 | 2.617 | 1.289 | 1.658 | 2.358 |
| • • | 1.645 | 1.960 | 2.576 | 1.282 | 1.645 | 2.327 |

Figure 6: Statistical table for Student's $t(r)$.

**Critical Values of the F Distribution**
($\alpha = .05$)

| df within | df between | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 12 | 24 | ∞ |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.68 | 4.53 | 4.37 |
| 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.00 | 3.84 | 3.67 |
| 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.57 | 3.41 | 3.23 |
| 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.28 | 3.12 | 2.93 |
| 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.07 | 2.90 | 2.71 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 2.91 | 2.74 | 2.54 |
| 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.79 | 2.61 | 2.41 |
| 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.69 | 2.51 | 2.30 |
| 13 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.60 | 2.42 | 2.21 |
| 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.53 | 2.35 | 2.13 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.48 | 2.29 | 2.07 |
| 16 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.42 | 2.24 | 2.01 |
| 17 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.38 | 2.19 | 1.96 |
| 18 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.34 | 2.15 | 1.92 |
| 19 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.31 | 2.11 | 1.88 |
| 20 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.28 | 2.08 | 1.84 |
| 21 | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.25 | 2.05 | 1.81 |
| 22 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.23 | 2.03 | 1.78 |
| 23 | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.20 | 2.01 | 1.76 |
| 24 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.18 | 1.98 | 1.73 |
| 25 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.16 | 1.96 | 1.71 |
| 26 | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.15 | 1.95 | 1.69 |
| 27 | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.13 | 1.93 | 1.67 |
| 28 | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.12 | 1.91 | 1.66 |
| 29 | 4.18 | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.10 | 1.90 | 1.64 |
| 30 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.09 | 1.89 | 1.62 |
| 40 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.00 | 1.79 | 1.51 |
| 60 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 1.92 | 1.70 | 1.39 |
| 80 | 3.96 | 3.11 | 2.72 | 2.49 | 2.33 | 2.21 | 2.13 | 2.06 | 1.88 | 1.65 | 1.33 |
| 100 | 3.94 | 3.09 | 2.70 | 2.46 | 2.31 | 2.19 | 2.10 | 2.03 | 1.85 | 1.63 | 1.28 |
| 120 | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.18 | 2.09 | 2.02 | 1.83 | 1.61 | 1.26 |
| ∞ | 3.84 | 3.00 | 2.61 | 2.37 | 2.22 | 2.10 | 2.01 | 1.94 | 1.75 | 1.52 | 1.00 |

Figure 7: Statistical table for $F(m, n)$ at the 5% level.

**Critical Values of the F Distribution**
($\alpha$ = .01)

| df | df between | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| within | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 12 | 24 | ∞ |
| 5 | 16.26 | 13.27 | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 | 9.89 | 9.47 | 9.02 |
| 6 | 13.75 | 10.92 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.72 | 7.31 | 6.88 |
| 7 | 12.25 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.47 | 6.07 | 5.65 |
| 8 | 11.26 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.67 | 5.28 | 4.86 |
| 9 | 10.56 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.11 | 4.73 | 4.31 |
| 10 | 10.04 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.71 | 4.33 | 3.91 |
| 11 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.40 | 4.02 | 3.60 |
| 12 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.16 | 3.78 | 3.36 |
| 13 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 3.96 | 3.59 | 3.17 |
| 14 | 8.86 | 6.51 | 5.56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 3.80 | 3.43 | 3.01 |
| 15 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.67 | 3.29 | 2.87 |
| 16 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.55 | 3.18 | 2.75 |
| 17 | 8.40 | 6.11 | 5.18 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.46 | 3.08 | 2.65 |
| 18 | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.37 | 3.00 | 2.57 |
| 19 | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.30 | 2.92 | 2.49 |
| 20 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.23 | 2.86 | 2.42 |
| 21 | 8.02 | 5.78 | 4.87 | 4.37 | 4.04 | 3.81 | 3.64 | 3.51 | 3.17 | 2.80 | 2.36 |
| 22 | 7.95 | 5.72 | 4.82 | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.12 | 2.75 | 2.31 |
| 23 | 7.88 | 5.66 | 4.76 | 4.26 | 3.94 | 3.71 | 3.54 | 3.41 | 3.07 | 2.70 | 2.26 |
| 24 | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.03 | 2.66 | 2.21 |
| 25 | 7.77 | 5.57 | 4.68 | 4.18 | 3.85 | 3.63 | 3.46 | 3.32 | 2.99 | 2.62 | 2.17 |
| 26 | 7.72 | 5.53 | 4.64 | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 2.96 | 2.58 | 2.13 |
| 27 | 7.68 | 5.49 | 4.60 | 4.11 | 3.78 | 3.56 | 3.39 | 3.26 | 2.93 | 2.55 | 2.10 |
| 28 | 7.64 | 5.45 | 4.57 | 4.07 | 3.75 | 3.53 | 3.36 | 3.23 | 2.90 | 2.52 | 2.07 |
| 29 | 7.60 | 5.42 | 4.54 | 4.04 | 3.73 | 3.50 | 3.33 | 3.20 | 2.87 | 2.49 | 2.04 |
| 30 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 2.84 | 2.47 | 2.01 |
| 40 | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.66 | 2.29 | 1.81 |
| 60 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.50 | 2.12 | 1.60 |
| 80 | 6.96 | 4.88 | 4.04 | 3.56 | 3.26 | 3.04 | 2.87 | 2.74 | 2.42 | 2.03 | 1.50 |
| 100 | 6.90 | 4.82 | 3.98 | 3.51 | 3.21 | 2.99 | 2.82 | 2.69 | 2.37 | 1.98 | 1.43 |
| 120 | 6.85 | 4.79 | 3.95 | 3.48 | 3.17 | 2.96 | 2.79 | 2.66 | 2.34 | 1.95 | 1.38 |
| ∞ | 6.64 | 4.61 | 3.78 | 3.32 | 3.02 | 2.80 | 2.64 | 2.51 | 2.19 | 1.79 | 1.00 |

Figure 8: Statistical table for $F(m, n)$ at the 1% level.

**Critical Values of the $\chi^2$ Distribution**

| df | Area in the Upper Tail | | | | | |
|---|---|---|---|---|---|---|
| | 0.99 | 0.95 | 0.9 | 0.1 | **0.05** | **0.01** |
| 1 | 0.000 | 0.004 | 0.016 | 2.706 | 3.841 | 6.635 |
| 2 | 0.020 | 0.103 | 0.211 | 4.605 | 5.991 | 9.210 |
| 3 | 0.115 | 0.352 | 0.584 | 6.251 | 7.815 | 11.345 |
| 4 | 0.297 | 0.711 | 1.064 | 7.779 | 9.488 | 13.277 |
| 5 | 0.554 | 1.145 | 1.610 | 9.236 | 11.070 | 15.086 |
| 6 | 0.872 | 1.635 | 2.204 | 10.645 | 12.592 | 16.812 |
| 7 | 1.239 | 2.167 | 2.833 | 12.017 | 14.067 | 18.475 |
| 8 | 1.646 | 2.733 | 3.490 | 13.362 | 15.507 | 20.090 |
| 9 | 2.088 | 3.325 | 4.168 | 14.684 | 16.919 | 21.666 |
| 10 | 2.558 | 3.940 | 4.865 | 15.987 | 18.307 | 23.209 |
| 11 | 3.053 | 4.575 | 5.578 | 17.275 | 19.675 | 24.725 |
| 12 | 3.571 | 5.226 | 6.304 | 18.549 | 21.026 | 26.217 |
| 13 | 4.107 | 5.892 | 7.042 | 19.812 | 22.362 | 27.688 |
| 14 | 4.660 | 6.571 | 7.790 | 21.064 | 23.685 | 29.141 |
| 15 | 5.229 | 7.261 | 8.547 | 22.307 | 24.996 | 30.578 |
| 16 | 5.812 | 7.962 | 9.312 | 23.542 | 26.296 | 32.000 |
| 17 | 6.408 | 8.672 | 10.085 | 24.769 | 27.587 | 33.409 |
| 18 | 7.015 | 9.390 | 10.865 | 25.989 | 28.869 | 34.805 |
| 19 | 7.633 | 10.117 | 11.651 | 27.204 | 30.144 | 36.191 |
| 20 | 8.260 | 10.851 | 12.443 | 28.412 | 31.410 | 37.566 |
| 21 | 8.897 | 11.591 | 13.240 | 29.615 | 32.671 | 38.932 |
| 22 | 9.542 | 12.338 | 14.041 | 30.813 | 33.924 | 40.289 |
| 23 | 10.196 | 13.091 | 14.848 | 32.007 | 35.172 | 41.638 |
| 24 | 10.856 | 13.848 | 15.659 | 33.196 | 36.415 | 42.980 |
| 25 | 11.524 | 14.611 | 16.473 | 34.382 | 37.652 | 44.314 |

Figure 9: Statistical table for $\chi^2(q)$.