IENAC22 / ECONOMETRICS 1 / APPLIED PROBLEM SET 2

Topic: Further multivariate regression

- This problem set is an introduction to the **programming features** of EViews 6 (or it can be completed using menu commands in EViews 12). The data is available on the website as `metals_data.txt`.
- The problem set covers (i) file import, (ii) workfile and series creation, (iii) graphical analysis, (iv) ordinary least squares, (v) testing of linear hypotheses, (vi) matrix manipulation, and (vii) storing and execution of EViews 6 program (`.prg`) files.
- We use production data for the U.S. metals industry. There are 27 statewide observations on (i) value added (output Q_i), (ii) labour input (L_i), and (iii) gross value of plant and equipment (capital stock K_i). The columns are ordered as (i), (ii), (iii).
- We specify a **Cobb-Douglas production function** $Q = Q(L, K)$:

  $$Q_i = \alpha_0^{\star} L_i^{\alpha_1} K_i^{\alpha_2}, \quad i = 1, 2, \ldots, n,$$

  from which:

  $$\ln(Q_i) = \alpha_0 + \alpha_1 \ln(L_i) + \alpha_2 \ln(K_i) + u_i$$

  is an econometric model.
- Follow the steps described in Figures **1–13**, and respond to the following questions:

  1. Regress log output on a constant, log labour input, and log capital input (**eq01**):

$$\widehat{\ln(Q_i)} \approx \boxed{[\qquad\quad]} + \boxed{[\qquad\quad]}\ln(L_i) + \boxed{[\qquad\quad]}\ln(K_i),$$

with standard errors $\boxed{[\quad]}$, $\boxed{[\quad]}$, and $\boxed{[\quad]}$ respectively. Check using t statistics whether the estimated coefficients are *individually* significant at the 99% level.

2. Test whether the **labour elasticity of output** ($\partial \ln(Q_i)/\partial \ln(L_i)$) is equal to 1, at the 95% level:

   $$H_0 : \boxed{[\quad]}, \quad H_1 : \boxed{[\quad]}$$

   with t statistic (first, give the t statistic in terms of $\widehat{\alpha}_j$; then substitute in the estimated value; finally, simplify):

   $$t = \frac{\boxed{[\quad]}}{\boxed{[\quad]}} \approx \frac{\boxed{[\quad]}}{\boxed{[\quad]}} \approx \boxed{[\quad]}$$

   Use the full available accuracy of the estimated coefficients when computing the results. Since (give the absolute computed value of t from above; compare this to the appropriate critical value $t_{crit}(n-k)$):

   $$|t| \approx \boxed{[\quad]} > / < t_{\boxed{[\quad]}}(\boxed{[\quad]}) \approx \boxed{[\quad]}$$

   we reject / do not reject the null hypothesis at the 95% level. Perform the same test using the F statistic (use $F(1, n-k) = t^2(n-k)$ to compute the F statistic; then compare this to the appropriate critical value $F_{crit}(1, n-k)$):

   $$\begin{array}{rcl} F(1, \boxed{[\quad]}) & := & t^2(\boxed{[\quad]}) \approx \boxed{[\quad]} \\ & & > / < \; F_{\boxed{[\quad]}}(1, \boxed{[\quad]}) \approx \boxed{[\quad]} \end{array}$$

   and so we reject / do not reject the null at the 95% level.

3. Test for **constant returns to scale** at the 95% level of significance (also, how would you define increasing and decreasing returns to scale?). Note that:

$$\begin{aligned} Q(\lambda L_i, \lambda K_i) &= \alpha_0(\lambda L_i)^{\alpha_1}(\lambda K_i)^{\alpha_2} \\ &= \alpha_0 \lambda^{\alpha_1} L_i^{\alpha_1} \lambda^{\alpha_2} K_i^{\alpha_2} \\ &= \lambda^{(\alpha_1+\alpha_2)} Q_i = \lambda Q_i \end{aligned}$$

if and only if $\alpha_1 + \alpha_2 = 1$. So, the hypothesis test of interest is:

$$H_0 : \boxed{[\qquad\quad]}, \quad H_1 : \boxed{[\qquad\quad]}$$

from which we can construct the F statistic (first, write this in terms of $\widehat{\alpha}_j$; and identify the distribution $F(q, n-k)$ that the F statistic follows):

$$F = \frac{\boxed{[\qquad\quad]}}{\boxed{[\qquad\quad]}} \sim F(\boxed{[\qquad\quad]}, \boxed{[\qquad\quad]})$$

We then need to compute both $\widehat{\sigma}^2$ and $\widehat{\mathrm{Var}}(\cdot)$, where

$$\widehat{\sigma}^2 = \frac{\widehat{u}'\widehat{u}}{n-k} \approx \boxed{[\qquad\quad]}$$

Then, $\widehat{\mathrm{Var}}(\widehat{\alpha}) = \widehat{\sigma}^2(X'X)^{-1}$, where $(X'X)^{-1} \approx$:

$$\begin{pmatrix} \boxed{[\qquad\quad]} & \boxed{[\qquad\quad]} & \boxed{[\qquad\quad]} \\ \boxed{[\qquad\quad]} & \boxed{[\qquad\quad]} & \boxed{[\qquad\quad]} \\ \boxed{[\qquad\quad]} & \boxed{[\qquad\quad]} & \boxed{[\qquad\quad]} \end{pmatrix}$$

Note that $\widehat{\text{Var}}(\widehat{\alpha}_1 + \widehat{\alpha}_2) = \widehat{\text{Var}}(\widehat{\alpha}_1) + \widehat{\text{Var}}(\widehat{\alpha}_2) + 2\widehat{\text{Cov}}(\widehat{\alpha}_1, \widehat{\alpha}_2)$. Then (substitute in the computed values, and simplify):

$$\widehat{\text{Var}}(\widehat{\alpha}_1 + \widehat{\alpha}_2) = \widehat{\sigma}^2(\boxed{[\qquad\qquad]} + \boxed{[\qquad\qquad]} + \boxed{[\qquad\qquad]}) \approx \boxed{[\qquad\qquad]}$$

So (compute the F statistic, and compare it to the appropriate critical value):

$$F \approx \boxed{[\qquad\qquad]} > / < F_{\boxed{[\qquad\qquad]}}(\boxed{[\qquad\qquad]}, \boxed{[\qquad\qquad]}) \approx \boxed{[\qquad\qquad]}$$

and so we reject / do not reject the null at the 95% level.

4. A generalization of the Cobb-Douglas model is the **translog model** (**eq02**):

$$\ln(Q_i) = \alpha_0 + \alpha_1 \ln(L_i) + \alpha_2 \ln(K_i) + \frac{1}{2}\alpha_3(\ln(L_i))^2 + \frac{1}{2}\alpha_4(\ln(K_i))^2 + \alpha_5 \ln(L_i)\ln(K_i) + u_i,$$

which reduces to the Cobb-Douglas model under $H_0 : \alpha_3 = \alpha_4 = \alpha_5 = 0$. Run the translog regression:

$$\begin{aligned}\widehat{\ln(Q_i)} &= \boxed{[\qquad\qquad]} + \boxed{[\qquad\qquad]}\ln(L_i) + \boxed{[\qquad\qquad]}\ln(K_i) \\ &+ \boxed{[\qquad\qquad]}(\ln(L_i))^2 + \boxed{[\qquad\qquad]}(\ln(K_i))^2 + \boxed{[\qquad\qquad]}\ln(L_i)\ln(K_i)\end{aligned}$$

EViews gives the "Wald" F statistic as:

$$F \approx \boxed{[\qquad\qquad]} > / < F_{\boxed{[\qquad\qquad]}}(\boxed{[\qquad\qquad]}, \boxed{[\qquad\qquad]}) \approx \boxed{[\qquad\qquad]}$$

and so we reject / do not reject the null hypothesis at the 95% level.

5. What is the estimated **capital elasticity of output** from the *translog* model?

$$\widehat{\eta} := \frac{\partial \widehat{\ln(Q_i)}}{\partial \ln(K_i)} \approx \boxed{[\qquad\quad]} + \boxed{[\qquad\quad]}\ln(K_i) + \boxed{[\qquad\quad]}\ln(L_i).$$

Replace $\ln(L_i)$ and $\ln(K_i)$ by their sample means $\frac{1}{n}\sum_{i=1}^{n}\ln(L_i) \approx \boxed{[\qquad\quad]}$ and $\frac{1}{n}\sum_{i=1}^{n}\ln(K_i) \approx \boxed{[\qquad\quad]}$, to give the capital elasticity of output evaluated at the sample means:

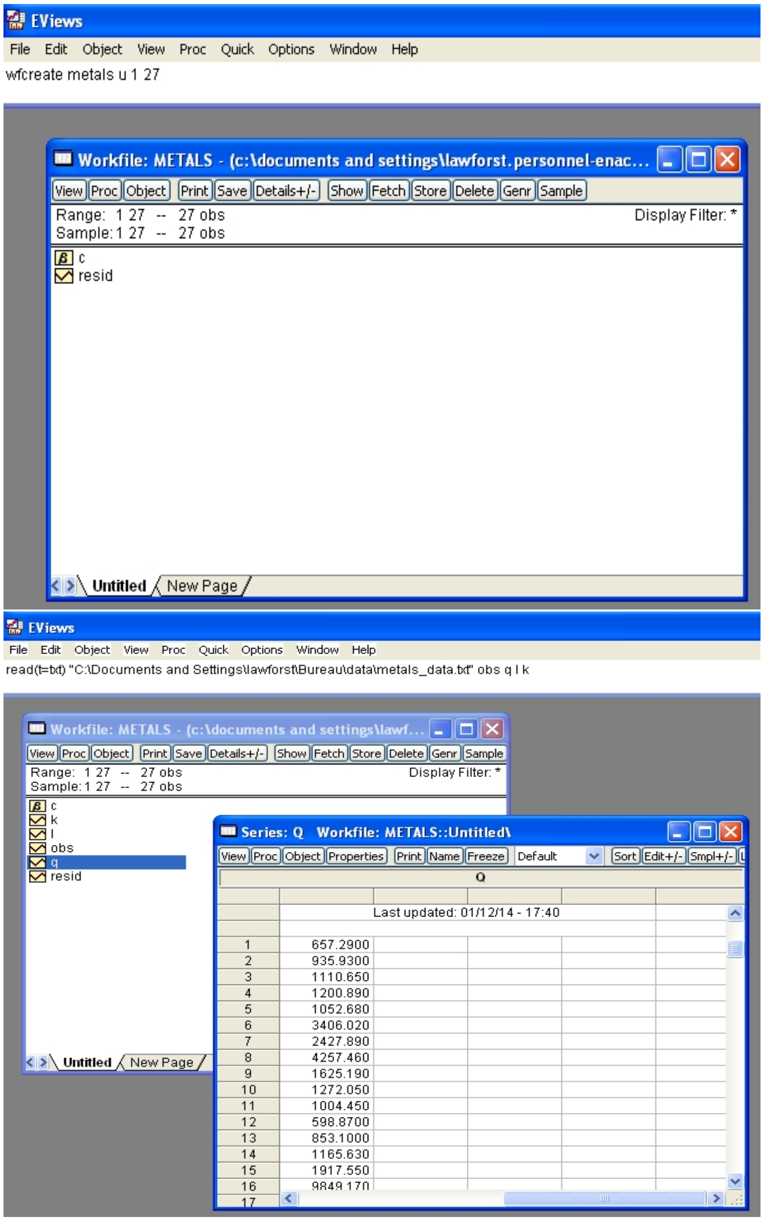
$$\widehat{\eta} \text{ at sample means } \approx \boxed{[\qquad\quad]}$$

so that a $\boxed{[\qquad\quad]}$% increase/decrease in capital results in a $\boxed{[\qquad\quad]}$% increase/decrease in output.

Importing Data into a Workfile

and Descriptive Statistics

RECORD THE COMMAND LINES IN A TEXT FILE AS YOU GO!



```
wfcreate metals u 1 27
```

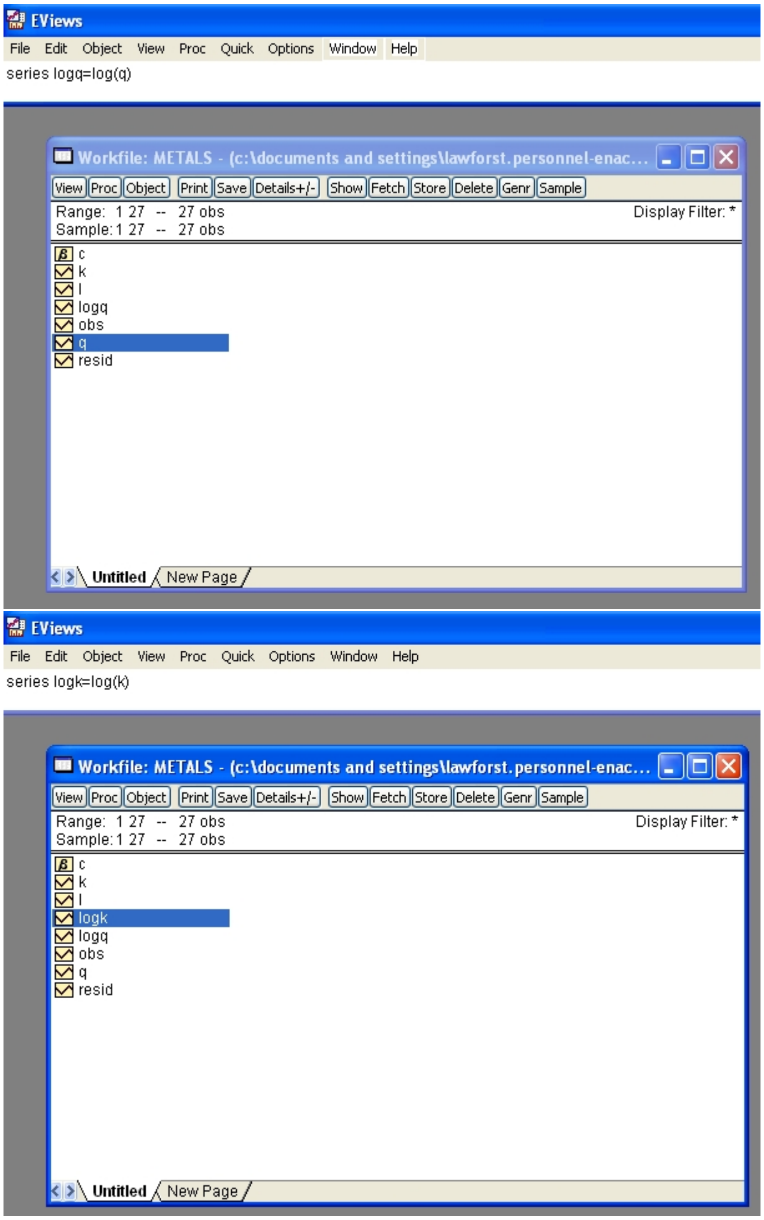
```
Range:  1 27  --  27 obs                    Display Filter: *
Sample: 1 27  --  27 obs
c
resid
```

```
read(t=txt) "C:\Documents and Settings\lawforst\Bureau\data\metals_data.txt" obs q l k
```

```
Range:  1 27  --  27 obs                    Display Filter: *
Sample: 1 27  --  27 obs
c
k
l
obs
q
resid
```

```
                    Q
        Last updated: 01/12/14 - 17:40

 1     657.2900
 2     935.9300
 3     1110.650
 4     1200.890
 5     1052.680
 6     3406.020
 7     2427.890
 8     4257.460
 9     1625.190
10     1272.050
11     1004.450
12     598.8700
13     853.1000
14     1165.630
15     1917.550
16     9849.170
17
```

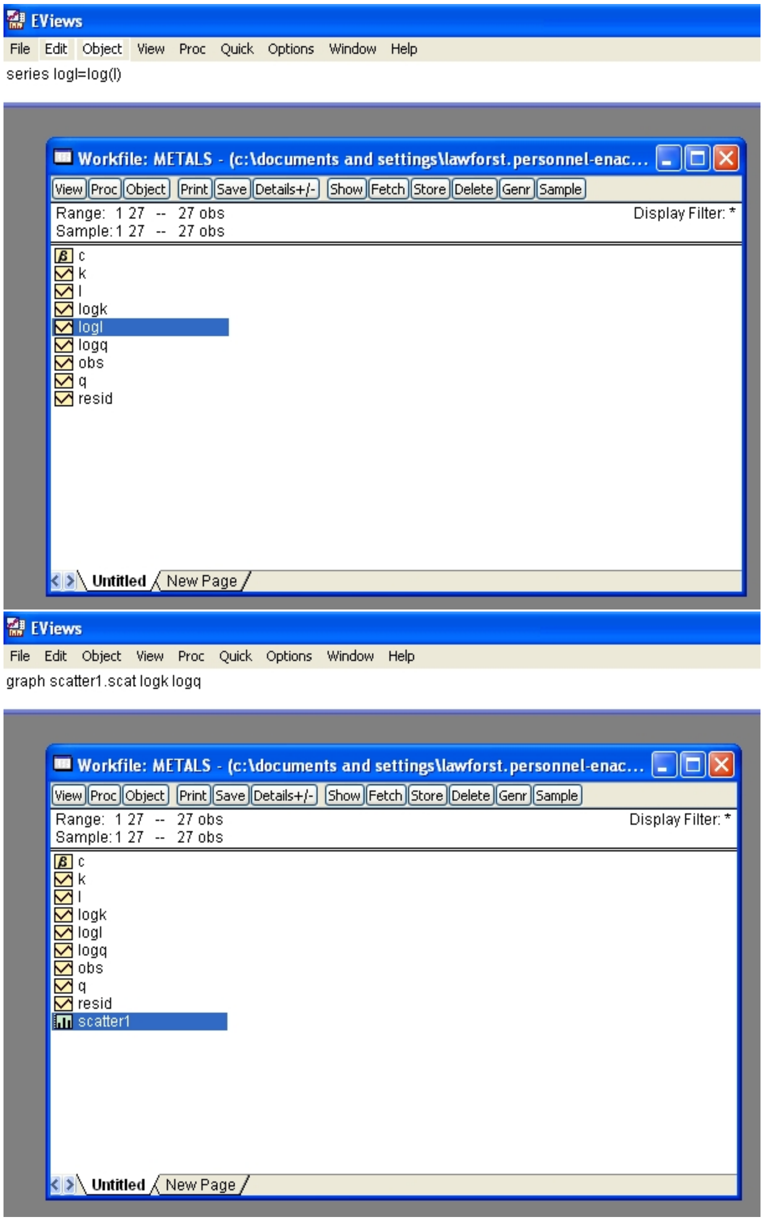
Figure 1: Use programming commands rather than menu-driven commands throughout this Problem Set, if possible. **See the top of each screen!** Create a workfile and import the datafile `metals_data.txt` into objects obs, q, l and k (output, labour and capital). **You will need to specify the path appropriately.**



```
series logq=log(q)
```

```
series logk=log(k)
```

Figure 2: Create the log-transformed series logq = log(q) and logk = log(k).



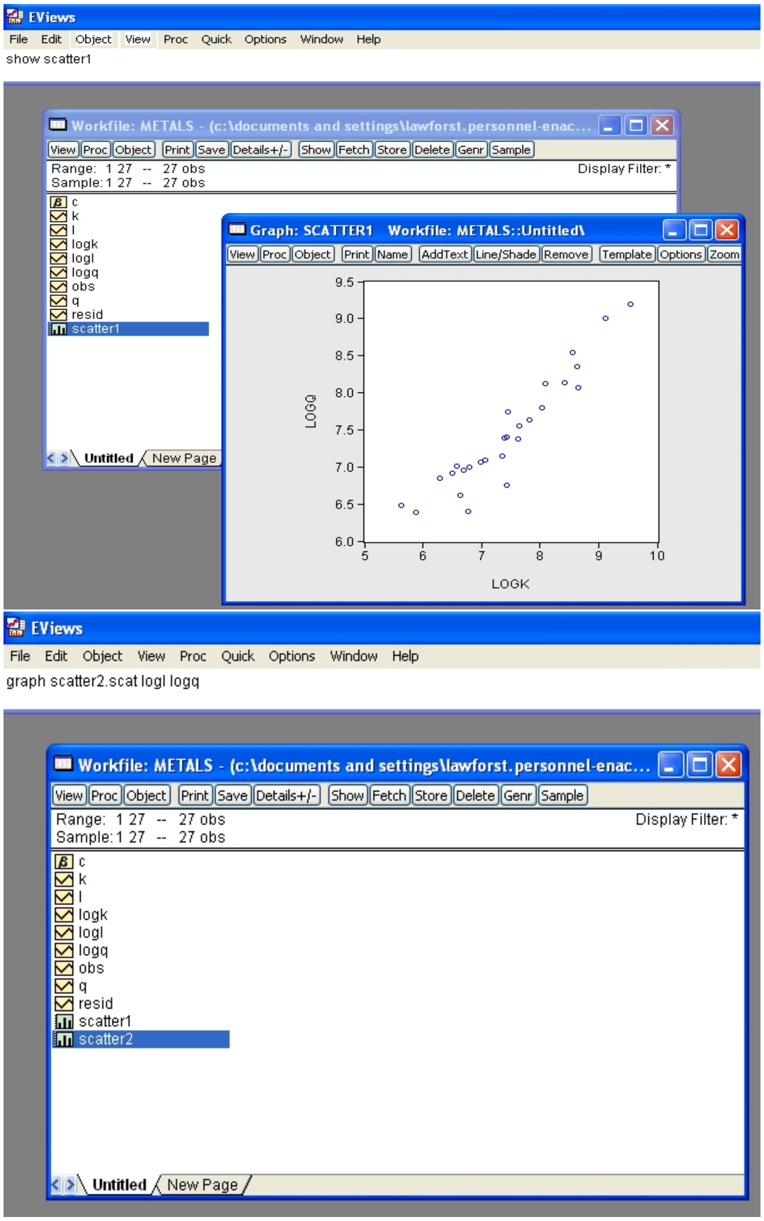
```
series logl=log(l)
```

Range: 1 27 -- 27 obs
Sample: 1 27 -- 27 obs

```
graph scatter1.scat logk logq
```

Range: 1 27 -- 27 obs
Sample: 1 27 -- 27 obs

Figure 3: Create a series logl = log(l), and create a scatterplot of logq against logk, and name the graph ‘scatter1’.



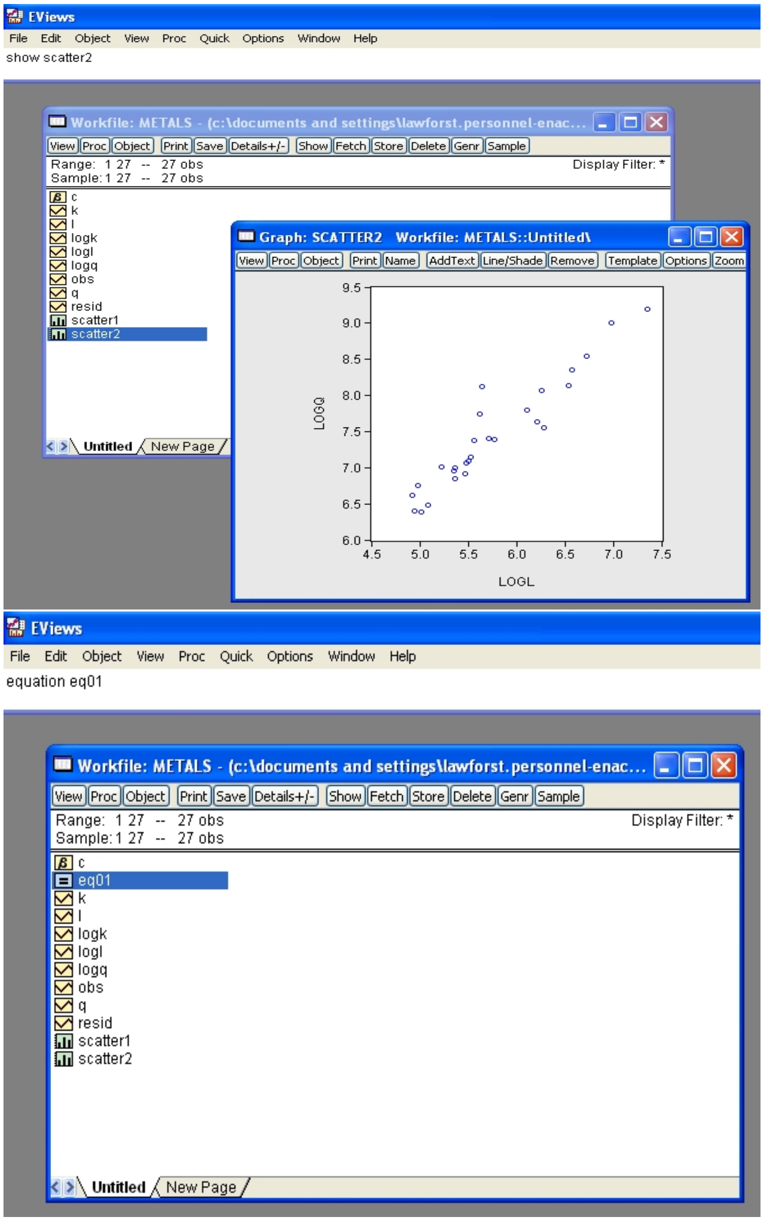
```
show scatter1
```

```
Range:  1 27  --  27 obs
Sample: 1 27  --  27 obs
```

```
graph scatter2.scat logl logq
```

```
Range:  1 27  --  27 obs
Sample: 1 27  --  27 obs
```

Figure 4: Display the scatterplot ‘scatter1’, and create a scatterplot of logq against logl, and name this graph ‘scatter2’.



```
show scatter2
```

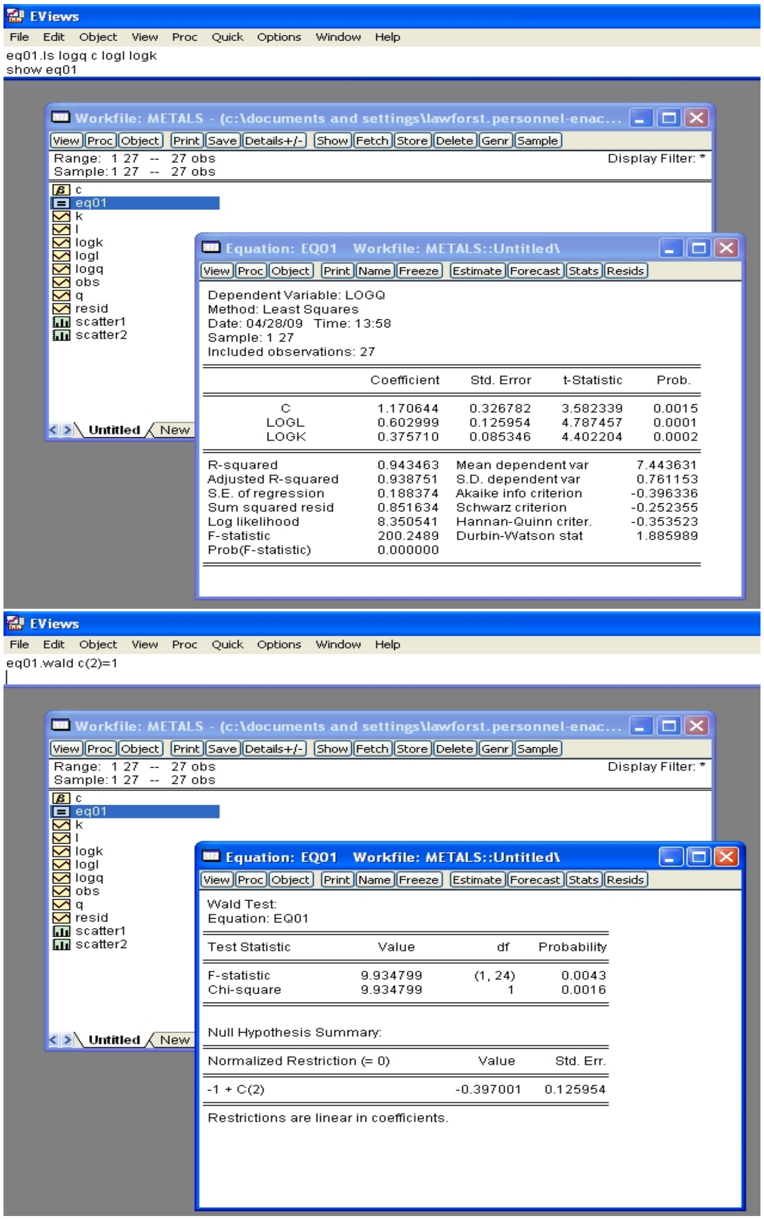
```
equation eq01
```

```
Range:  1 27  --  27 obs
Sample: 1 27  --  27 obs
```

Figure 5: Display the scatterplot ‘scatter2’. Create an equation *object* eq01. Note that the model has not yet been specified.

A First Regression

and Hypothesis Testing



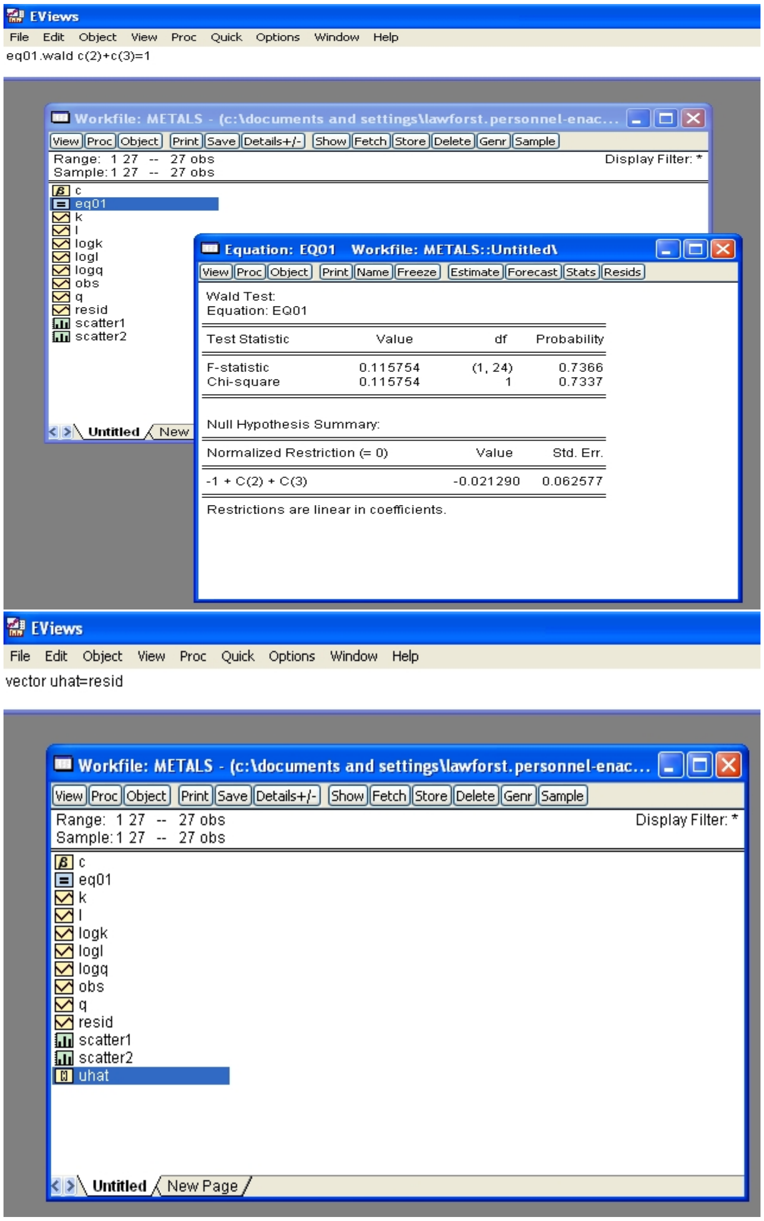
```
eq01.ls logq c logl logk
show eq01
```

```
Dependent Variable: LOGQ
Method: Least Squares
Date: 04/28/09   Time: 13:58
Sample: 1 27
Included observations: 27

                         Coefficient    Std. Error    t-Statistic     Prob.

            C             1.170644      0.326782      3.582339      0.0015
          LOGL            0.602999      0.125954      4.787457      0.0001
          LOGK            0.375710      0.085346      4.402204      0.0002

R-squared                 0.943463    Mean dependent var          7.443631
Adjusted R-squared        0.938751    S.D. dependent var          0.761153
S.E. of regression        0.188374    Akaike info criterion      -0.396336
Sum squared resid         0.851634    Schwarz criterion          -0.252355
Log likelihood            8.350541    Hannan-Quinn criter.       -0.353523
F-statistic               200.2489    Durbin-Watson stat          1.885989
Prob(F-statistic)         0.000000
```

```
eq01.wald c(2)=1
```

```
Wald Test:
Equation: EQ01

Test Statistic            Value            df      Probability

F-statistic             9.934799       (1, 24)          0.0043
Chi-square              9.934799             1          0.0016


Null Hypothesis Summary:

Normalized Restriction (= 0)               Value       Std. Err.

-1 + C(2)                              -0.397001        0.125954

Restrictions are linear in coefficients.
```

Figure 6: For eq01, run the Cobb-Douglas regression of logq on a constant, logl and logk, using ordinary least squares. Perform the hypothesis test (Wald) of $H_0 : \alpha_1 = 1$ against $H_1 : \alpha_1 \neq 1$.



```
eq01.wald c(2)+c(3)=1
```

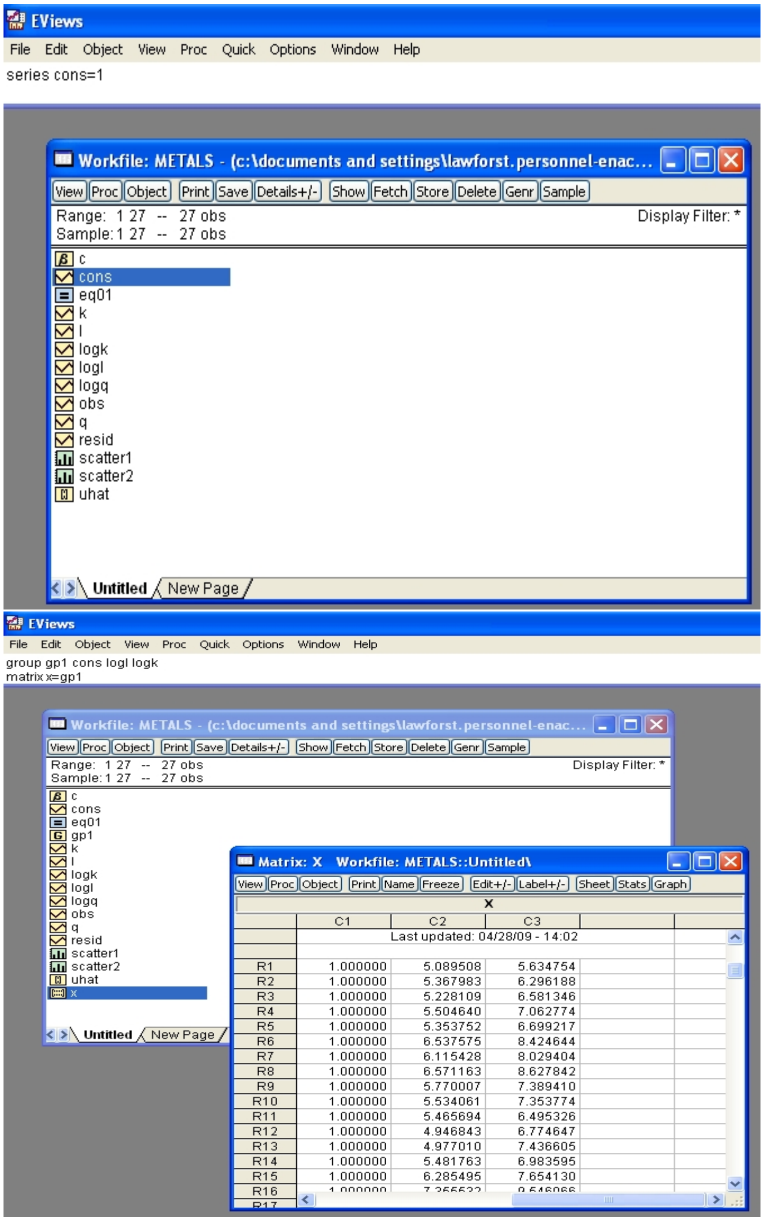
```
Range:  1 27  --  27 obs
Sample: 1 27  --  27 obs

c
eq01
k
l
logk
logl
logq
obs
q
resid
scatter1
scatter2
```

```
Wald Test:
Equation: EQ01

Test Statistic        Value         df        Probability

F-statistic           0.115754      (1, 24)   0.7366
Chi-square            0.115754      1         0.7337

Null Hypothesis Summary:

Normalized Restriction (= 0)        Value       Std. Err.

-1 + C(2) + C(3)                    -0.021290   0.062577

Restrictions are linear in coefficients.
```

```
vector uhat=resid
```

```
Range:  1 27  --  27 obs
Sample: 1 27  --  27 obs

c
eq01
k
l
logk
logl
logq
obs
q
resid
scatter1
scatter2
uhat
```

Figure 7: For eq01, perform the test (Wald) of $H_0 : \alpha_1 + \alpha_2 = 1$ against $H_1 : \alpha_1 + \alpha_2 \neq 1$. Create a series 'uhat' equal to the current estimated residual vector 'resid'.



```
series cons=1
```

Range: 1 27 -- 27 obs
Sample: 1 27 -- 27 obs
Display Filter: *

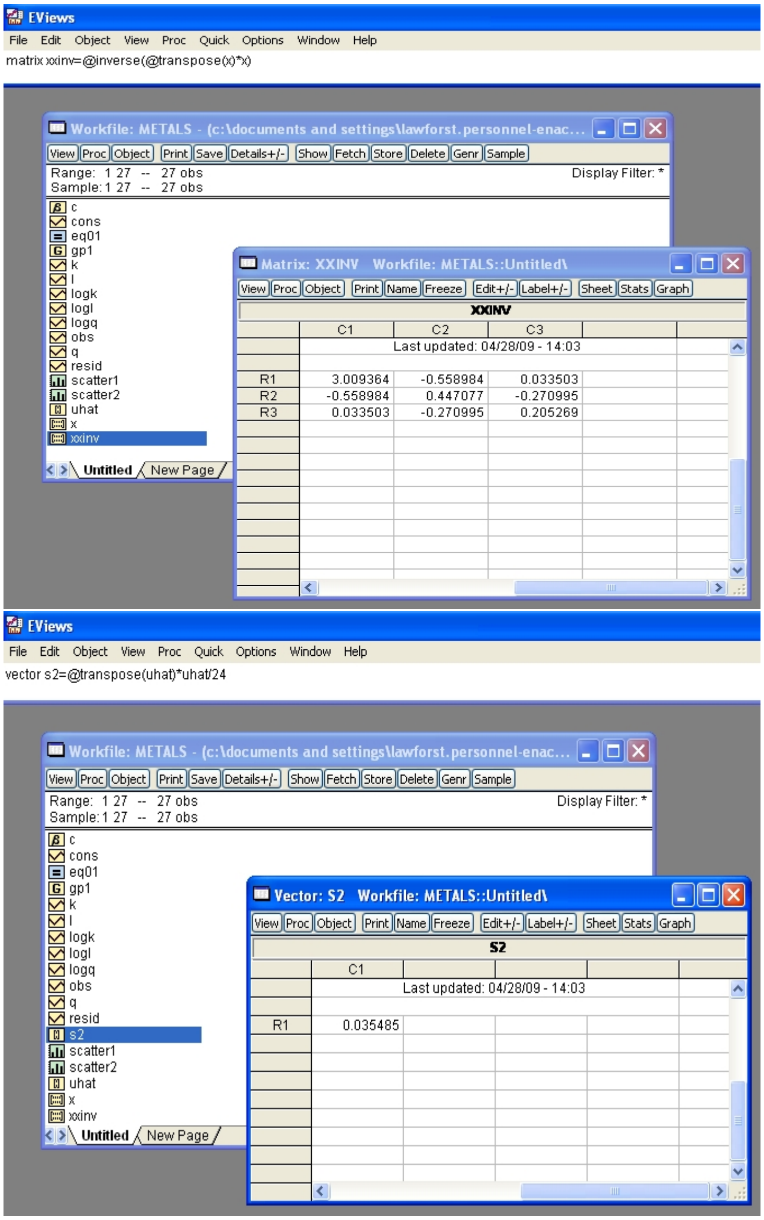
```
group gp1 cons logl logk
matrix x=gp1
```

X

Last updated: 04/28/09 - 14:02

	C1	C2	C3
R1	1.000000	5.089508	5.634754
R2	1.000000	5.367983	6.296188
R3	1.000000	5.228109	6.581346
R4	1.000000	5.504640	7.062774
R5	1.000000	5.353752	6.699217
R6	1.000000	6.537575	8.424644
R7	1.000000	6.115428	8.029404
R8	1.000000	6.571163	8.627842
R9	1.000000	5.770007	7.389410
R10	1.000000	5.534061	7.353774
R11	1.000000	5.465694	6.495326
R12	1.000000	4.946843	6.774647
R13	1.000000	4.977010	7.436605
R14	1.000000	5.481763	6.983595
R15	1.000000	6.285495	7.654130

Figure 8: Create a 27 × 1 vector of ones, ‘cons’. Create a group, named ‘gp1’, containing cons, logl and logk. Create a matrix object, named x, containing group gp1.



```
matrix xxinv=@inverse(@transpose(x)*x)
```

XXINV

	C1	C2	C3
R1	3.009364	-0.558984	0.033503
R2	-0.558984	0.447077	-0.270995
R3	0.033503	-0.270995	0.205269

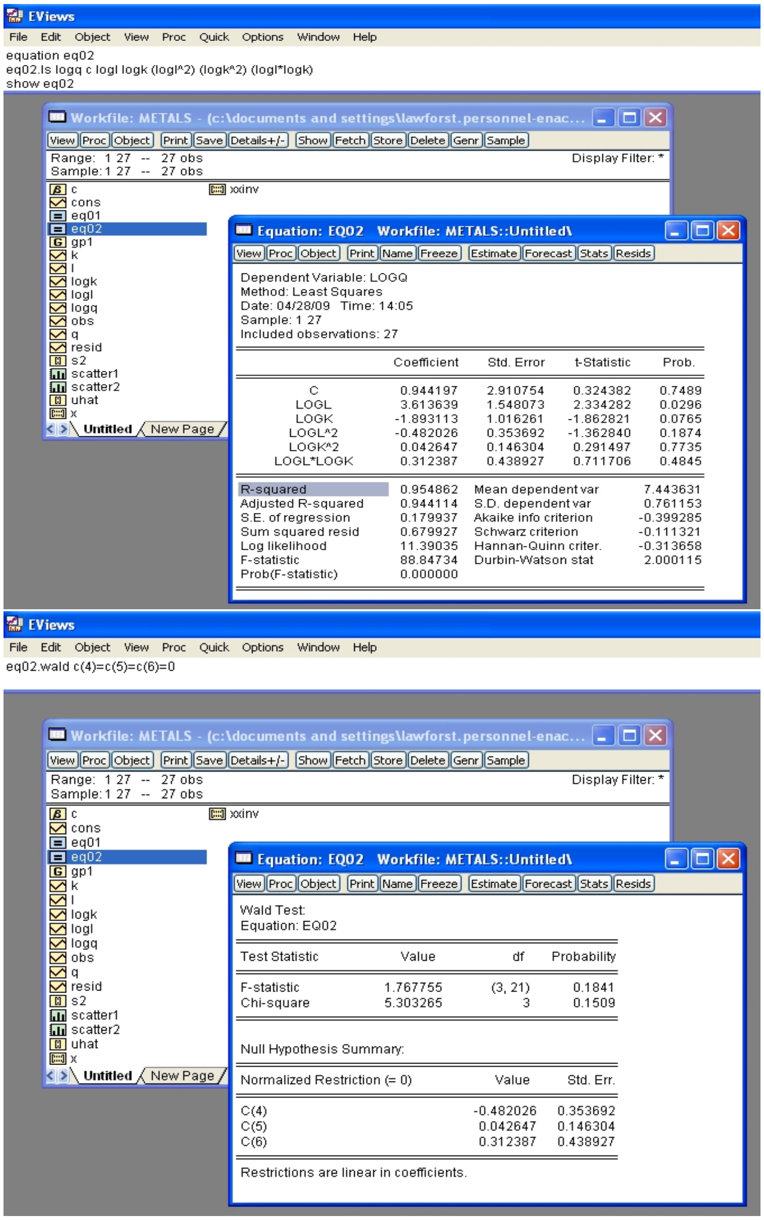
```
vector s2=@transpose(uhat)*uhat/24
```

S2

	C1
R1	0.035485

Figure 9: Compute $(X'X)^{-1}$ directly from X. Compute $\widehat{\sigma}^2 = \widehat{u}'\widehat{u}/(n-k)$ directly from uhat. This can be used directly to give $\widehat{\text{Var}}(\widehat{\alpha}) = \widehat{\sigma}^2(X'X)^{-1}$.

A Second Regression and Hypothesis Testing



```
equation eq02
eq02.ls logq c logl logk (logl^2) (logk^2) (logl*logk)
show eq02
```

```
Dependent Variable: LOGQ
Method: Least Squares
Date: 04/28/09   Time: 14:05
Sample: 1 27
Included observations: 27

                    Coefficient   Std. Error   t-Statistic   Prob.

          C          0.944197     2.910754      0.324382     0.7489
        LOGL         3.613639     1.548073      2.334282     0.0296
        LOGK        -1.893113     1.016261     -1.862821     0.0765
       LOGL^2       -0.482026     0.353692     -1.362840     0.1874
       LOGK^2        0.042647     0.146304      0.291497     0.7735
     LOGL*LOGK       0.312387     0.438927      0.711706     0.4845

R-squared             0.954862   Mean dependent var       7.443631
Adjusted R-squared    0.944114   S.D. dependent var       0.761153
S.E. of regression    0.179937   Akaike info criterion   -0.399285
Sum squared resid     0.679927   Schwarz criterion       -0.111321
Log likelihood        11.39035   Hannan-Quinn criter.    -0.313658
F-statistic           88.84734   Durbin-Watson stat       2.000115
Prob(F-statistic)     0.000000
```

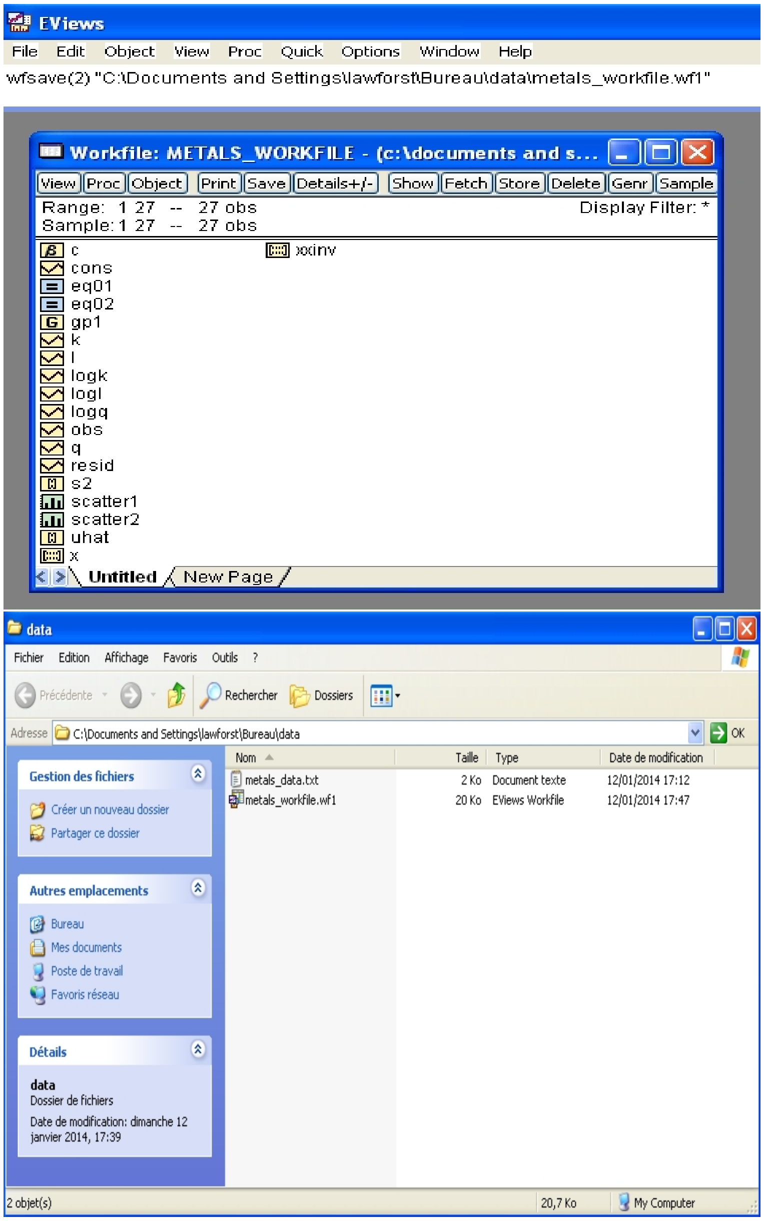
```
eq02.wald c(4)=c(5)=c(6)=0
```

```
Wald Test:
Equation: EQ02

Test Statistic         Value         df      Probability

F-statistic          1.767755      (3, 21)      0.1841
Chi-square           5.303265         3         0.1509


Null Hypothesis Summary:

Normalized Restriction (= 0)            Value     Std. Err.

C(4)                                -0.482026     0.353692
C(5)                                 0.042647     0.146304
C(6)                                 0.312387     0.438927

Restrictions are linear in coefficients.
```

Figure 10: Create an equation object eq02, and run the translog regression of logq on a constant, logl, logk, logl squared, logk squared, and logl×logk, using ordinary least squares. Perform the test (Wald) of $H_0 : \alpha_3 = \alpha_4 = \alpha_5 = 0$ against $H_1 :$ not H_0.

Saving the Workfile

and Running a Simple Program



```
wfsave(2) "C:\Documents and Settings\lawforst\Bureau\data\metals_workfile.wf1"
```

Workfile: METALS_WORKFILE - (c:\documents and s...

```
Range:  1 27   --   27 obs
Sample: 1 27   --   27 obs
```

Display Filter: *

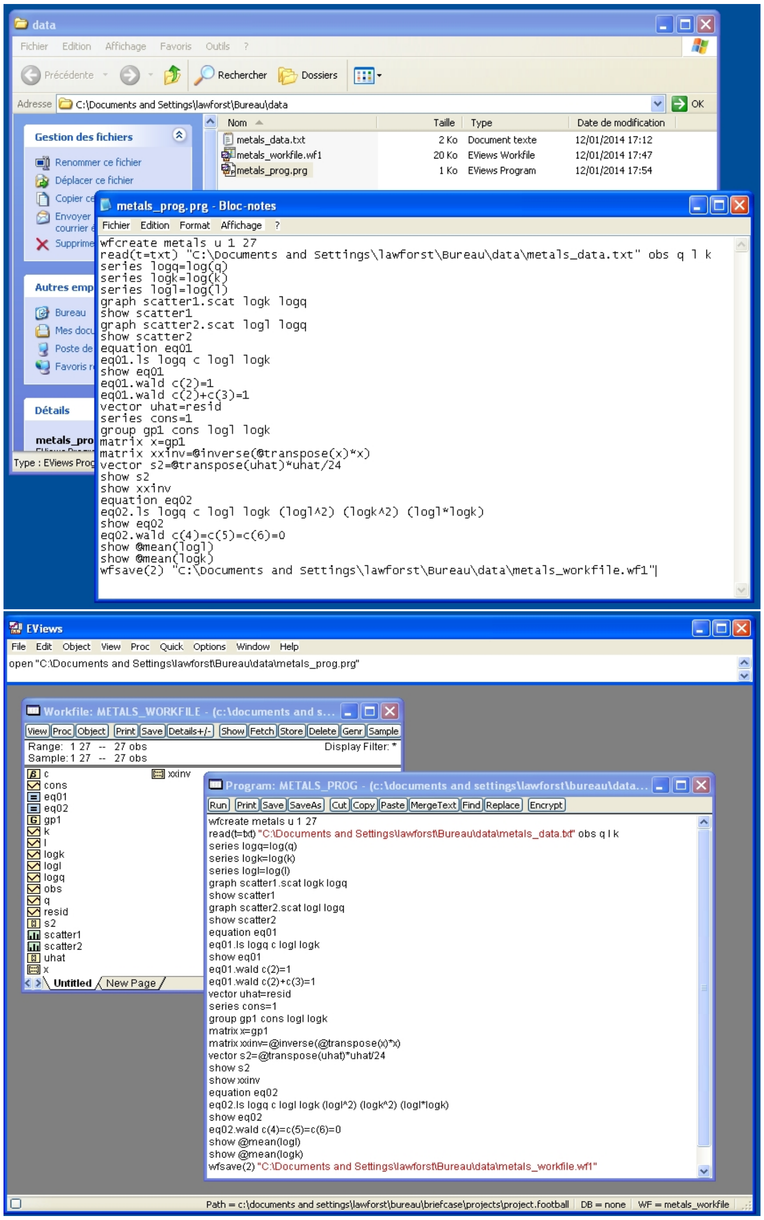
C:\Documents and Settings\lawforst\Bureau\data

Nom	Taille	Type	Date de modification
metals_data.txt	2 Ko	Document texte	12/01/2014 17:12
metals_workfile.wf1	20 Ko	EViews Workfile	12/01/2014 17:47

data
Dossier de fichiers
Date de modification: dimanche 12 janvier 2014, 17:39

2 objet(s) 20,7 Ko My Computer

Figure 11: Save the workfile as ‘metals_workfile.wf1’, **with an appropriate path**. Copy all of the above EViews commands into a text file `metals_prog.prg`



Adresse C:\Documents and Settings\lawforst\Bureau\data

Nom	Taille	Type	Date de modification
metals_data.txt	2 Ko	Document texte	12/01/2014 17:12
metals_workfile.wf1	20 Ko	EViews Workfile	12/01/2014 17:47
metals_prog.prg	1 Ko	EViews Program	12/01/2014 17:54

```
wfcreate metals u 1 27
read(t=txt) "C:\Documents and Settings\lawforst\Bureau\data\metals_data.txt" obs q l k
series logq=log(q)
series logk=log(k)
series logl=log(l)
graph scatter1.scat logk logq
show scatter1
graph scatter2.scat logl logq
show scatter2
equation eq01
eq01.ls logq c logl logk
show eq01
eq01.wald c(2)=1
eq01.wald c(2)+c(3)=1
vector uhat=resid
series cons=1
group gp1 cons logl logk
matrix x=gp1
matrix xxinv=@inverse(@transpose(x)*x)
vector s2=@transpose(uhat)*uhat/24
show s2
show xxinv
equation eq02
eq02.ls logq c logl logk (logl^2) (logk^2) (logl*logk)
show eq02
eq02.wald c(4)=c(5)=c(6)=0
show @mean(logl)
show @mean(logk)
wfsave(2) "C:\Documents and Settings\lawforst\Bureau\data\metals_workfile.wf1"
```

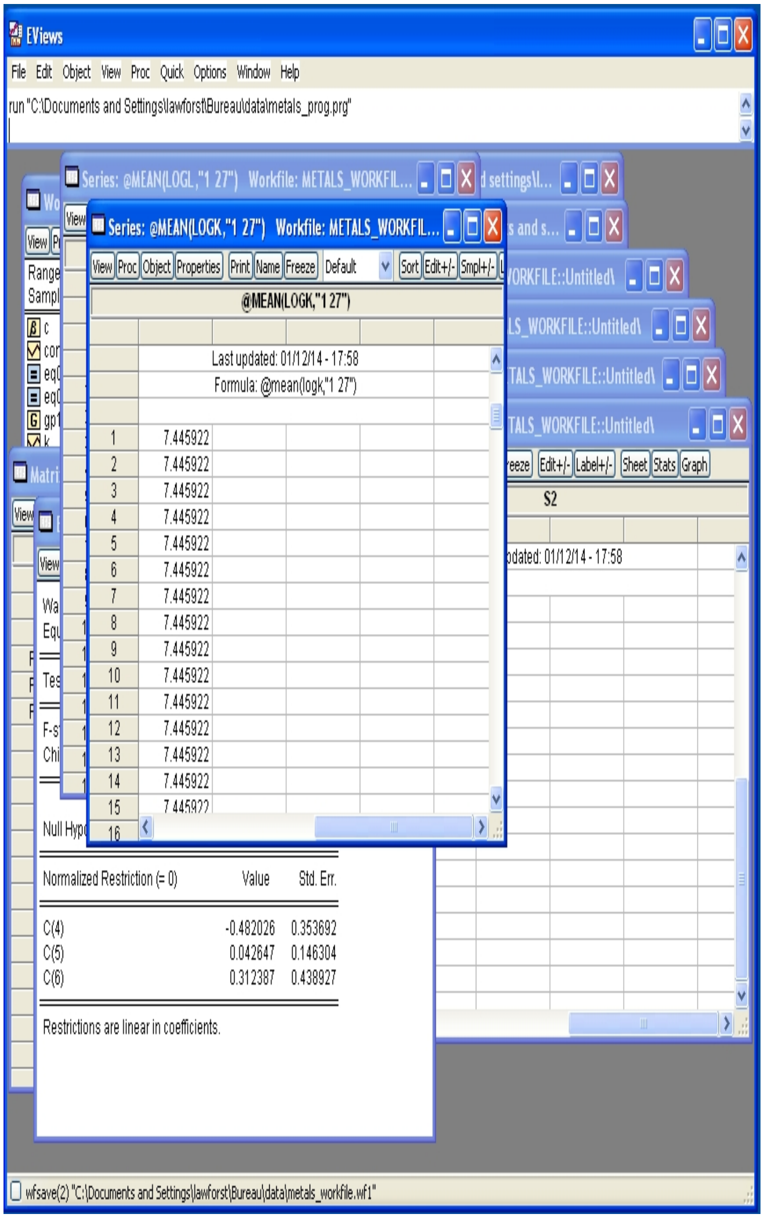
```
open "C:\Documents and Settings\lawforst\Bureau\data\metals_prog.prg"
```

Range: 1 27 -- 27 obs Display Filter: *
Sample: 1 27 -- 27 obs

wfcreate metals u 1 27
read(t=txt) "C:\Documents and Settings\lawforst\Bureau\data\metals_data.txt" obs q l k
series logq=log(q)
series logk=log(k)
series logl=log(l)
graph scatter1.scat logk logq
show scatter1
graph scatter2.scat logl logq
show scatter2
equation eq01
eq01.ls logq c logl logk
show eq01
eq01.wald c(2)=1
eq01.wald c(2)+c(3)=1
vector uhat=resid
series cons=1
group gp1 cons logl logk
matrix x=gp1
matrix xxinv=@inverse(@transpose(x)*x)
vector s2=@transpose(uhat)*uhat/24
show s2
show xxinv
equation eq02
eq02.ls logq c logl logk (logl^2) (logk^2) (logl*logk)
show eq02
eq02.wald c(4)=c(5)=c(6)=0
show @mean(logl)
show @mean(logk)
wfsave(2) "C:\Documents and Settings\lawforst\Bureau\data\metals_workfile.wf1"

Path = c:\documents and settings\lawforst\bureau\briefcase\projects\project.football DB = none WF = metals_workfile

Figure 12: EViews 6 code `metals_prog.prg`, opened in Notepad. Code can be opened in EViews 6, with the ‘open’ command. **Note that four additional lines of code have been added here (lines 21, 22, 27 and 28) / you should add these too.**



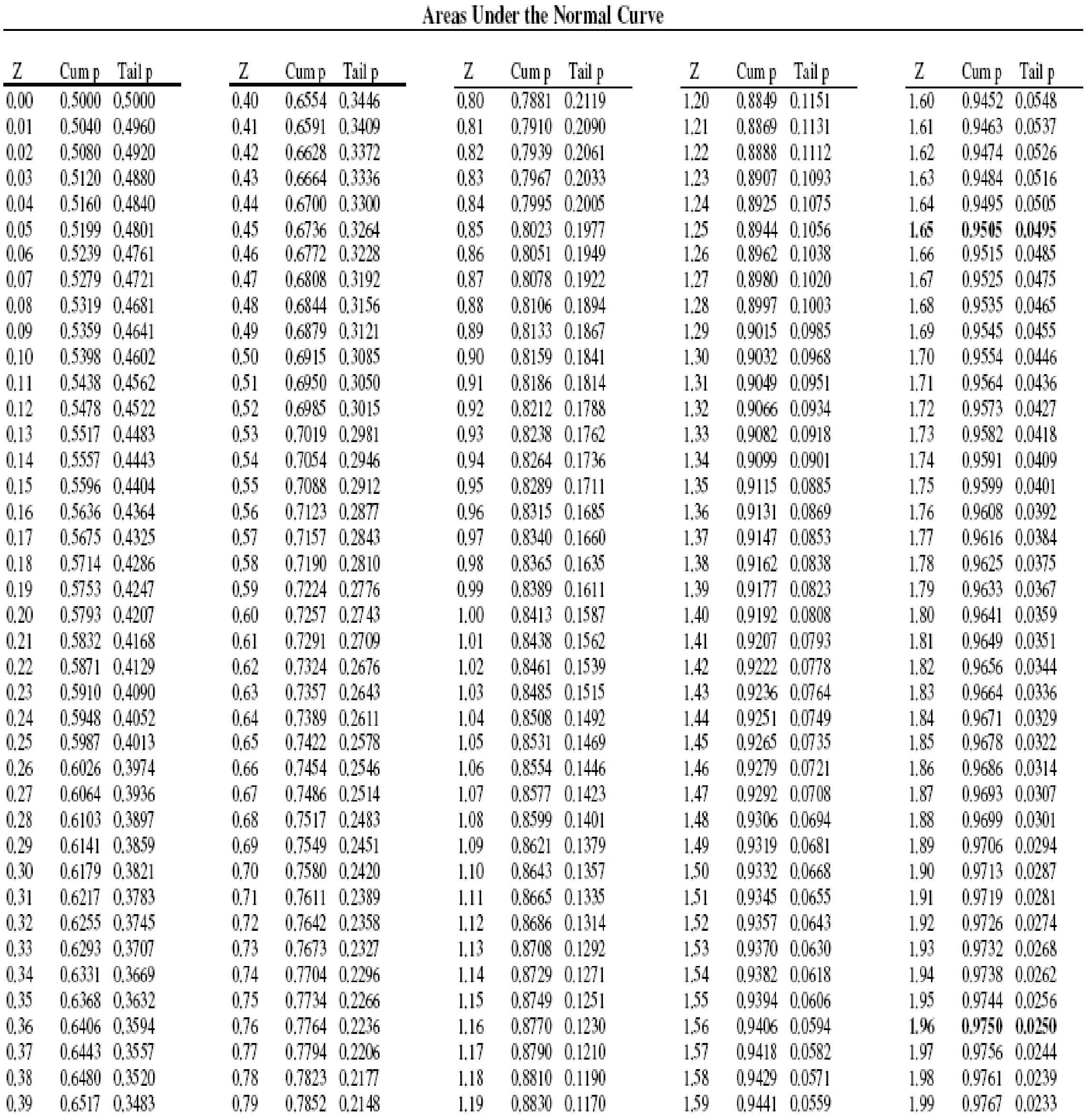
```
run "C:\Documents and Settings\lawforst\Bureau\data\metals_prog.prg"
```

```
@MEAN(LOGK,"1 27")

     Last updated: 01/12/14 - 17:58
     Formula: @mean(logk,"1 27")

1    7.445922
2    7.445922
3    7.445922
4    7.445922
5    7.445922
6    7.445922
7    7.445922
8    7.445922
9    7.445922
10   7.445922
11   7.445922
12   7.445922
13   7.445922
14   7.445922
15   7.445922
16
```

```
Normalized Restriction (= 0)        Value       Std. Err.

C(4)                             -0.482026      0.353692
C(5)                              0.042647      0.146304
C(6)                              0.312387      0.438927

Restrictions are linear in coefficients.
```

```
wfsave(2) "C:\Documents and Settings\lawforst\Bureau\data\metals_workfile.wf1"
```

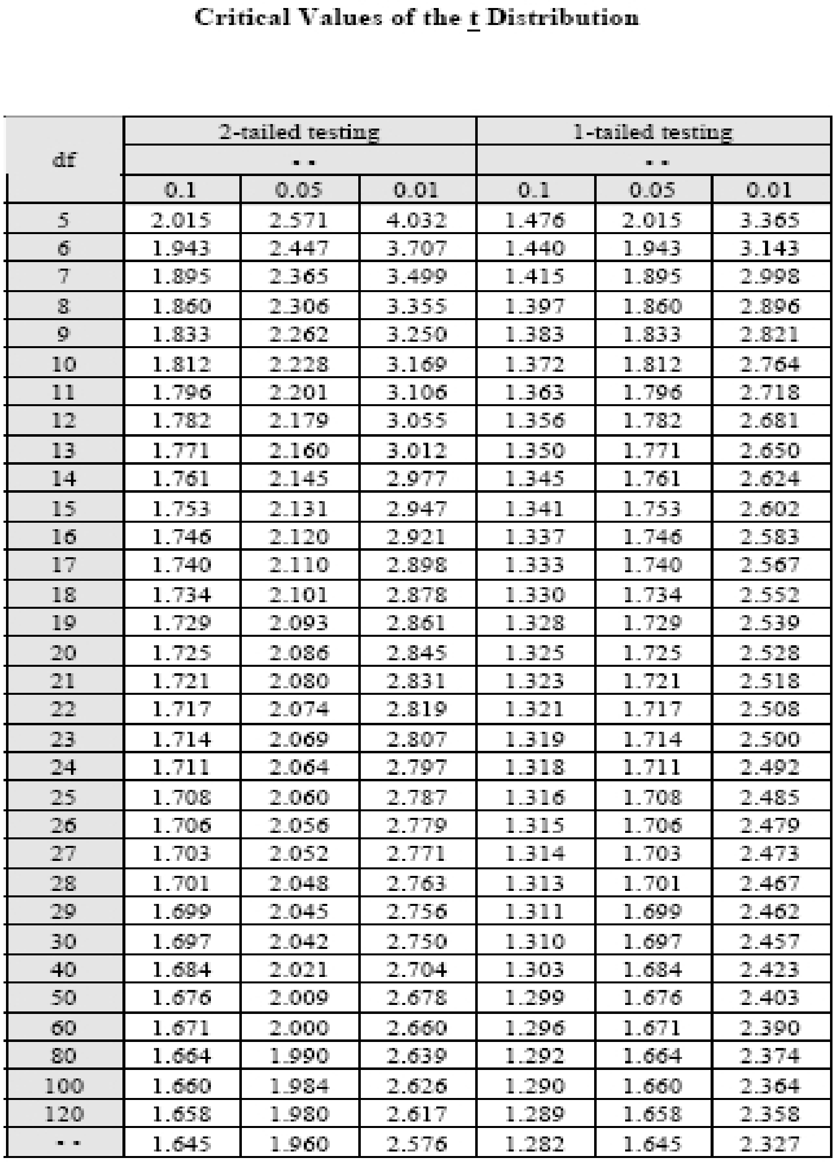
Figure 13: Close the open workfile (there is no need to save this). The code can then be run within EViews 6 using the 'run' command, or by clicking on the 'Run' button. This procedure is very useful in practice since it enables a (long) sequence of commands to be executed repeatedly, e.g. with modified data, or with modified models.



Areas Under the Normal Curve

Z	Cum p	Tail p	Z	Cum p	Tail p	Z	Cum p	Tail p	Z	Cum p	Tail p	Z	Cum p	Tail p
0.00	0.5000	0.5000	0.40	0.6554	0.3446	0.80	0.7881	0.2119	1.20	0.8849	0.1151	1.60	0.9452	0.0548
0.01	0.5040	0.4960	0.41	0.6591	0.3409	0.81	0.7910	0.2090	1.21	0.8869	0.1131	1.61	0.9463	0.0537
0.02	0.5080	0.4920	0.42	0.6628	0.3372	0.82	0.7939	0.2061	1.22	0.8888	0.1112	1.62	0.9474	0.0526
0.03	0.5120	0.4880	0.43	0.6664	0.3336	0.83	0.7967	0.2033	1.23	0.8907	0.1093	1.63	0.9484	0.0516
0.04	0.5160	0.4840	0.44	0.6700	0.3300	0.84	0.7995	0.2005	1.24	0.8925	0.1075	1.64	0.9495	0.0505
0.05	0.5199	0.4801	0.45	0.6736	0.3264	0.85	0.8023	0.1977	1.25	0.8944	0.1056	**1.65**	**0.9505**	**0.0495**
0.06	0.5239	0.4761	0.46	0.6772	0.3228	0.86	0.8051	0.1949	1.26	0.8962	0.1038	1.66	0.9515	0.0485
0.07	0.5279	0.4721	0.47	0.6808	0.3192	0.87	0.8078	0.1922	1.27	0.8980	0.1020	1.67	0.9525	0.0475
0.08	0.5319	0.4681	0.48	0.6844	0.3156	0.88	0.8106	0.1894	1.28	0.8997	0.1003	1.68	0.9535	0.0465
0.09	0.5359	0.4641	0.49	0.6879	0.3121	0.89	0.8133	0.1867	1.29	0.9015	0.0985	1.69	0.9545	0.0455
0.10	0.5398	0.4602	0.50	0.6915	0.3085	0.90	0.8159	0.1841	1.30	0.9032	0.0968	1.70	0.9554	0.0446
0.11	0.5438	0.4562	0.51	0.6950	0.3050	0.91	0.8186	0.1814	1.31	0.9049	0.0951	1.71	0.9564	0.0436
0.12	0.5478	0.4522	0.52	0.6985	0.3015	0.92	0.8212	0.1788	1.32	0.9066	0.0934	1.72	0.9573	0.0427
0.13	0.5517	0.4483	0.53	0.7019	0.2981	0.93	0.8238	0.1762	1.33	0.9082	0.0918	1.73	0.9582	0.0418
0.14	0.5557	0.4443	0.54	0.7054	0.2946	0.94	0.8264	0.1736	1.34	0.9099	0.0901	1.74	0.9591	0.0409
0.15	0.5596	0.4404	0.55	0.7088	0.2912	0.95	0.8289	0.1711	1.35	0.9115	0.0885	1.75	0.9599	0.0401
0.16	0.5636	0.4364	0.56	0.7123	0.2877	0.96	0.8315	0.1685	1.36	0.9131	0.0869	1.76	0.9608	0.0392
0.17	0.5675	0.4325	0.57	0.7157	0.2843	0.97	0.8340	0.1660	1.37	0.9147	0.0853	1.77	0.9616	0.0384
0.18	0.5714	0.4286	0.58	0.7190	0.2810	0.98	0.8365	0.1635	1.38	0.9162	0.0838	1.78	0.9625	0.0375
0.19	0.5753	0.4247	0.59	0.7224	0.2776	0.99	0.8389	0.1611	1.39	0.9177	0.0823	1.79	0.9633	0.0367
0.20	0.5793	0.4207	0.60	0.7257	0.2743	1.00	0.8413	0.1587	1.40	0.9192	0.0808	1.80	0.9641	0.0359
0.21	0.5832	0.4168	0.61	0.7291	0.2709	1.01	0.8438	0.1562	1.41	0.9207	0.0793	1.81	0.9649	0.0351
0.22	0.5871	0.4129	0.62	0.7324	0.2676	1.02	0.8461	0.1539	1.42	0.9222	0.0778	1.82	0.9656	0.0344
0.23	0.5910	0.4090	0.63	0.7357	0.2643	1.03	0.8485	0.1515	1.43	0.9236	0.0764	1.83	0.9664	0.0336
0.24	0.5948	0.4052	0.64	0.7389	0.2611	1.04	0.8508	0.1492	1.44	0.9251	0.0749	1.84	0.9671	0.0329
0.25	0.5987	0.4013	0.65	0.7422	0.2578	1.05	0.8531	0.1469	1.45	0.9265	0.0735	1.85	0.9678	0.0322
0.26	0.6026	0.3974	0.66	0.7454	0.2546	1.06	0.8554	0.1446	1.46	0.9279	0.0721	1.86	0.9686	0.0314
0.27	0.6064	0.3936	0.67	0.7486	0.2514	1.07	0.8577	0.1423	1.47	0.9292	0.0708	1.87	0.9693	0.0307
0.28	0.6103	0.3897	0.68	0.7517	0.2483	1.08	0.8599	0.1401	1.48	0.9306	0.0694	1.88	0.9699	0.0301
0.29	0.6141	0.3859	0.69	0.7549	0.2451	1.09	0.8621	0.1379	1.49	0.9319	0.0681	1.89	0.9706	0.0294
0.30	0.6179	0.3821	0.70	0.7580	0.2420	1.10	0.8643	0.1357	1.50	0.9332	0.0668	1.90	0.9713	0.0287
0.31	0.6217	0.3783	0.71	0.7611	0.2389	1.11	0.8665	0.1335	1.51	0.9345	0.0655	1.91	0.9719	0.0281
0.32	0.6255	0.3745	0.72	0.7642	0.2358	1.12	0.8686	0.1314	1.52	0.9357	0.0643	1.92	0.9726	0.0274
0.33	0.6293	0.3707	0.73	0.7673	0.2327	1.13	0.8708	0.1292	1.53	0.9370	0.0630	1.93	0.9732	0.0268
0.34	0.6331	0.3669	0.74	0.7704	0.2296	1.14	0.8729	0.1271	1.54	0.9382	0.0618	1.94	0.9738	0.0262
0.35	0.6368	0.3632	0.75	0.7734	0.2266	1.15	0.8749	0.1251	1.55	0.9394	0.0606	1.95	0.9744	0.0256
0.36	0.6406	0.3594	0.76	0.7764	0.2236	1.16	0.8770	0.1230	1.56	0.9406	0.0594	**1.96**	**0.9750**	**0.0250**
0.37	0.6443	0.3557	0.77	0.7794	0.2206	1.17	0.8790	0.1210	1.57	0.9418	0.0582	1.97	0.9756	0.0244
0.38	0.6480	0.3520	0.78	0.7823	0.2177	1.18	0.8810	0.1190	1.58	0.9429	0.0571	1.98	0.9761	0.0239
0.39	0.6517	0.3483	0.79	0.7852	0.2148	1.19	0.8830	0.1170	1.59	0.9441	0.0559	1.99	0.9767	0.0233

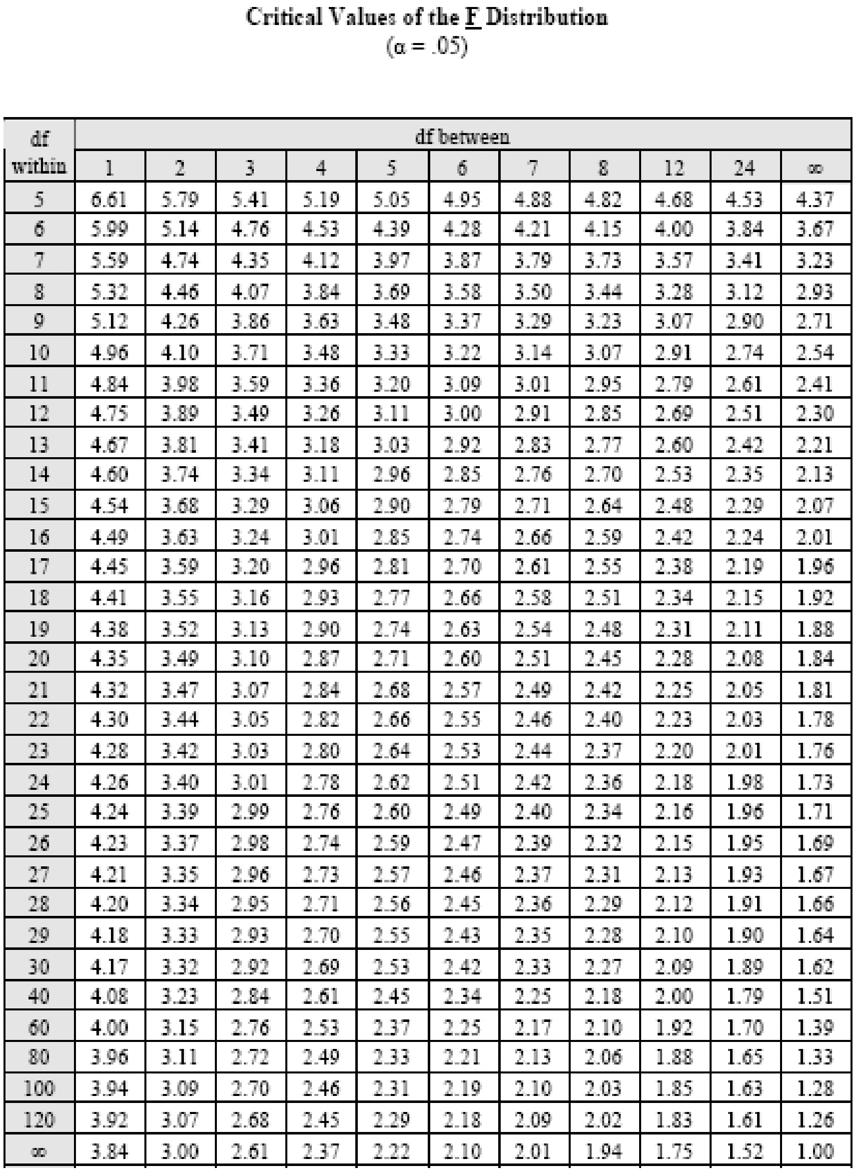
Figure 14: Statistical table for N(0, 1).



Critical Values of the t Distribution

df	2-tailed testing			1-tailed testing		
	• •			• •		
	0.1	0.05	0.01	0.1	0.05	0.01
5	2.015	2.571	4.032	1.476	2.015	3.365
6	1.943	2.447	3.707	1.440	1.943	3.143
7	1.895	2.365	3.499	1.415	1.895	2.998
8	1.860	2.306	3.355	1.397	1.860	2.896
9	1.833	2.262	3.250	1.383	1.833	2.821
10	1.812	2.228	3.169	1.372	1.812	2.764
11	1.796	2.201	3.106	1.363	1.796	2.718
12	1.782	2.179	3.055	1.356	1.782	2.681
13	1.771	2.160	3.012	1.350	1.771	2.650
14	1.761	2.145	2.977	1.345	1.761	2.624
15	1.753	2.131	2.947	1.341	1.753	2.602
16	1.746	2.120	2.921	1.337	1.746	2.583
17	1.740	2.110	2.898	1.333	1.740	2.567
18	1.734	2.101	2.878	1.330	1.734	2.552
19	1.729	2.093	2.861	1.328	1.729	2.539
20	1.725	2.086	2.845	1.325	1.725	2.528
21	1.721	2.080	2.831	1.323	1.721	2.518
22	1.717	2.074	2.819	1.321	1.717	2.508
23	1.714	2.069	2.807	1.319	1.714	2.500
24	1.711	2.064	2.797	1.318	1.711	2.492
25	1.708	2.060	2.787	1.316	1.708	2.485
26	1.706	2.056	2.779	1.315	1.706	2.479
27	1.703	2.052	2.771	1.314	1.703	2.473
28	1.701	2.048	2.763	1.313	1.701	2.467
29	1.699	2.045	2.756	1.311	1.699	2.462
30	1.697	2.042	2.750	1.310	1.697	2.457
40	1.684	2.021	2.704	1.303	1.684	2.423
50	1.676	2.009	2.678	1.299	1.676	2.403
60	1.671	2.000	2.660	1.296	1.671	2.390
80	1.664	1.990	2.639	1.292	1.664	2.374
100	1.660	1.984	2.626	1.290	1.660	2.364
120	1.658	1.980	2.617	1.289	1.658	2.358
• •	1.645	1.960	2.576	1.282	1.645	2.327

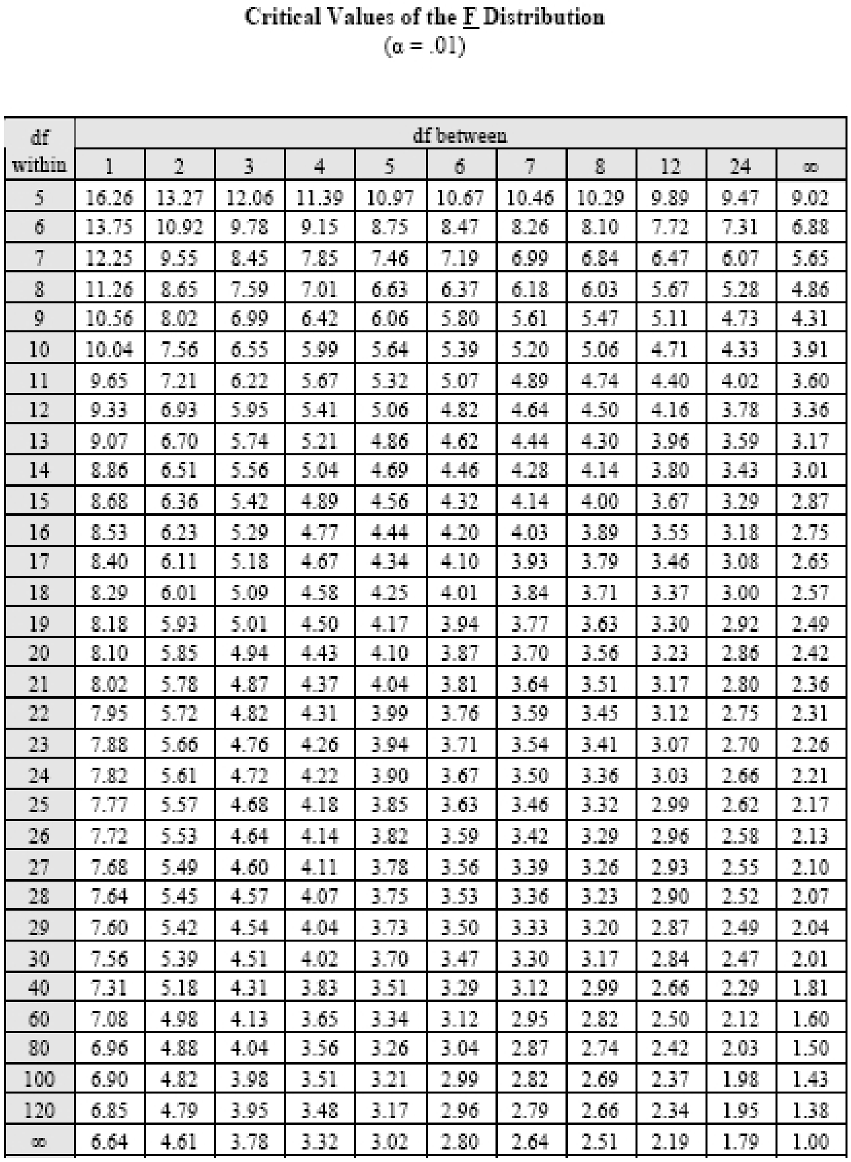
Figure 15: Statistical table for Student's $t(r)$



Critical Values of the F Distribution
(α = .05)

df	df between										
within	1	2	3	4	5	6	7	8	12	24	∞
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.68	4.53	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.00	3.84	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.57	3.41	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.28	3.12	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.07	2.90	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	2.91	2.74	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.79	2.61	2.41
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.69	2.51	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.60	2.42	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.53	2.35	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.48	2.29	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.42	2.24	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.38	2.19	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.34	2.15	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.31	2.11	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.28	2.08	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.25	2.05	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.23	2.03	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.20	2.01	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.18	1.98	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.16	1.96	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.15	1.95	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.13	1.93	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.12	1.91	1.66
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.10	1.90	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.09	1.89	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.00	1.79	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	1.92	1.70	1.39
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	1.88	1.65	1.33
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.85	1.63	1.28
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.83	1.61	1.26
∞	3.84	3.00	2.61	2.37	2.22	2.10	2.01	1.94	1.75	1.52	1.00

Figure 16: Statistical table for $F(m,n)$ at the 5% level

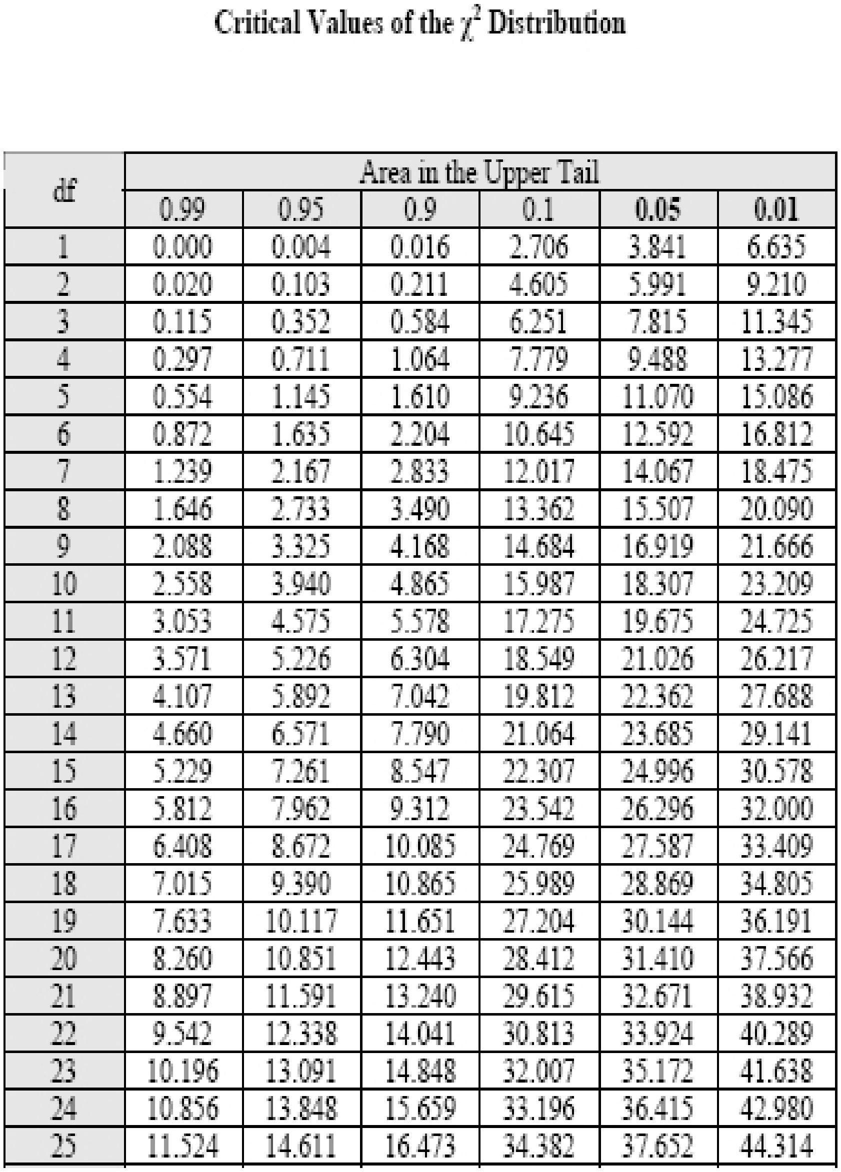


Critical Values of the F Distribution

(α = .01)

df within	df between										
	1	2	3	4	5	6	7	8	12	24	∞
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	9.89	9.47	9.02
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.72	7.31	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.47	6.07	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.67	5.28	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.11	4.73	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.71	4.33	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.40	4.02	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.16	3.78	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	3.96	3.59	3.17
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	3.80	3.43	3.01
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.67	3.29	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.55	3.18	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.46	3.08	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.37	3.00	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.30	2.92	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.23	2.86	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.17	2.80	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.12	2.75	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.07	2.70	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.03	2.66	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	2.99	2.62	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	2.96	2.58	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	2.93	2.55	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	2.90	2.52	2.07
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	2.87	2.49	2.04
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	2.84	2.47	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.66	2.29	1.81
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.50	2.12	1.60
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.42	2.03	1.50
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.37	1.98	1.43
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.34	1.95	1.38
∞	6.64	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.19	1.79	1.00

Figure 17: Statistical table for $F(m, n)$ at the 1% level



Critical Values of the χ^2 Distribution

df	Area in the Upper Tail					
	0.99	0.95	0.9	0.1	**0.05**	**0.01**
1	0.000	0.004	0.016	2.706	3.841	6.635
2	0.020	0.103	0.211	4.605	5.991	9.210
3	0.115	0.352	0.584	6.251	7.815	11.345
4	0.297	0.711	1.064	7.779	9.488	13.277
5	0.554	1.145	1.610	9.236	11.070	15.086
6	0.872	1.635	2.204	10.645	12.592	16.812
7	1.239	2.167	2.833	12.017	14.067	18.475
8	1.646	2.733	3.490	13.362	15.507	20.090
9	2.088	3.325	4.168	14.684	16.919	21.666
10	2.558	3.940	4.865	15.987	18.307	23.209
11	3.053	4.575	5.578	17.275	19.675	24.725
12	3.571	5.226	6.304	18.549	21.026	26.217
13	4.107	5.892	7.042	19.812	22.362	27.688
14	4.660	6.571	7.790	21.064	23.685	29.141
15	5.229	7.261	8.547	22.307	24.996	30.578
16	5.812	7.962	9.312	23.542	26.296	32.000
17	6.408	8.672	10.085	24.769	27.587	33.409
18	7.015	9.390	10.865	25.989	28.869	34.805
19	7.633	10.117	11.651	27.204	30.144	36.191
20	8.260	10.851	12.443	28.412	31.410	37.566
21	8.897	11.591	13.240	29.615	32.671	38.932
22	9.542	12.338	14.041	30.813	33.924	40.289
23	10.196	13.091	14.848	32.007	35.172	41.638
24	10.856	13.848	15.659	33.196	36.415	42.980
25	11.524	14.611	16.473	34.382	37.652	44.314

Figure 18: Statistical table for $\chi^2(q)$