ECONOMETRICS 1 / APPLIED PROBLEM SET 2

Topic: Further multivariate regression

- This problem set is an introduction to the **programming features** of EViews 6. The data is available on the website as `metals_data.txt`.

- The problem set covers (i) file import, (ii) workfile and series creation, (iii) graphical analysis, (iv) ordinary least squares, (v) testing of linear hypotheses, (vi) matrix manipulation, and (vii) storing and execution of EViews 6 program (`.prg`) files.

- We use production data for the U.S. metals industry. There are 27 statewide observations on (i) value added (output Q_i), (ii) labour input (L_i), and (iii) gross value of plant and equipment (capital stock K_i). The columns are ordered as (i), (ii), (iii). $n=27$

- We specify a **Cobb-Douglas production function** $Q = Q(L, K)$: assume that $L \geq 0,\ K \geq 0$

$$Q_i = \alpha_0^* L_i^{\alpha_1} K_i^{\alpha_2}, \quad i = 1, 2, \ldots, n,$$

from which:

$$\ln(Q_i) = \alpha_0 + \alpha_1 \ln(L_i) + \alpha_2 \ln(K_i) + u_i$$

"loglinear" model

is an econometric model.

- Follow the steps described in Figures **1–13**, and respond to the following questions:

1. Regress log output on a constant, log labour input, and log capital input (**eq01**):

$$\widehat{\ln(Q_i)} \approx \underbrace{\boxed{[\ 1.17\ ]}}_{\hat{\alpha}_0} + \underbrace{\boxed{[\ 0.60\ ]}}_{\hat{\alpha}_1} \ln(L_i) + \underbrace{\boxed{[\ 0.38\ ]}}_{\hat{\alpha}_2} \ln(K_i),$$

all coeffs. individually significant (Prob < 0.01)
at 1% level: $H_0: \alpha_j = 0$ $\quad H_1: \alpha_j \neq 0$ $\quad t_j = \dfrac{\hat{\alpha}_j}{\hat{se}(\hat{\alpha}_j)} \overset{H_0}{\approx} t(n-k)$
$\hat{Var}(\hat{\alpha}) = \hat{\sigma}^2 (X'X)^{-1}$; $\hat{\sigma}^2 = \dfrac{\hat{u}'\hat{u}}{n-k}$

with standard errors $\underset{}{\overset{\hat{se}(\hat{\alpha}_0)}{[\ 0.33\ ]}}$, $\overset{\hat{se}(\hat{\alpha}_1)}{[\ 0.13\ ]}$, and $\overset{\hat{se}(\hat{\alpha}_2)}{[\ 0.085\ ]}$ respectively. Check using t statistics whether the estimated coefficients are *individually* significant at the 99% level.

2. Test whether the **labour elasticity of output** $(\partial \ln(Q_i)/\partial \ln(L_i))$ is equal to 1, at the 95% level:

$$H_0 : [\ \alpha_1 = 1\ ], \quad H_1 : [\ \alpha_1 \neq 1\ ]$$

with t statistic (first, give the t statistic in terms of $\hat{\alpha}_j$; then substitute in the estimated value; finally, simplify):

$$t = \frac{[\ \hat{\alpha}_1 - 1\ ]}{[\ \hat{se}(\hat{\alpha}_1)\ ]} \approx \frac{[\ 0.60 - 1\ ]}{[\ 0.13\ ]} \approx [\ -3.15\ ]$$

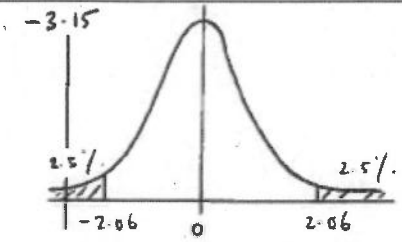
Use the full available accuracy of the estimated coefficients when computing the results. Since (give the absolute computed value of t from above; compare this to the appropriate critical value $t_{crit}(n-k)$):

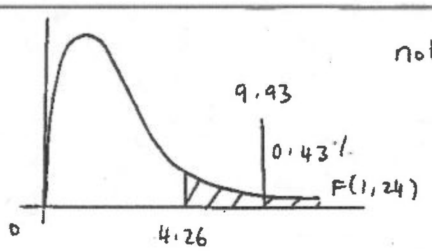
$$|t| \approx [\ 3.15\ ]\ > / \not>\ t_{[\ 0.975\ ]}([\ 24\ ]) \approx [\ 2.06\ ]$$

we reject ~~/ do not reject~~ the null hypothesis at the 95% level. Perform the same test using the F statistic (use $F(1, n-k) = t^2(n-k)$ to compute the F statistic; then compare this to the appropriate critical value $F_{crit}(1, n-k)$):

$$F(1, [\ 24\ ]) \quad := \quad t^2([\ 24\ ]) \approx [\ 9.93\ ]$$
$$> / \not>\ F_{[\ 0.95\ ]}(1, [\ 24\ ]) \approx [\ 4.26\ ]$$

and so we reject ~~/ do not reject~~ the null at the 95% level.





note: Prob = 0.0043 < 0.05

3. Test for **constant returns to scale** at the 95% level of significance (also, how would you define increasing and decreasing returns to scale?). Note that:

constant returns to scale
$Q(\lambda L, \lambda K) = \lambda Q$
increasing returns to scale
$Q(\lambda L, \lambda K) > \lambda Q$
decreasing returns to scale
$Q(\lambda L, \lambda K) < \lambda Q$

$$\begin{aligned} Q(\lambda L_i, \lambda K_i) &= \alpha_0^*(\lambda L_i)^{\alpha_1}(\lambda K_i)^{\alpha_2} \\ &= \alpha_0^*\lambda^{\alpha_1} L_i^{\alpha_1} \lambda^{\alpha_2} K_i^{\alpha_2} \\ &= \lambda^{(\alpha_1+\alpha_2)} Q_i = \lambda Q_i \end{aligned}$$

if and only if $\alpha_1 + \alpha_2 = 1$. So, the hypothesis test of interest is:

$$H_0 : \boxed{\alpha_1 + \alpha_2 = 1}, \quad H_1 : \boxed{\alpha_1 + \alpha_2 \neq 1}$$

$q = 1$ restriction under H_0.

from which we can construct the F statistic (first, write this in terms of $\widehat{\alpha}_j$; and identify the distribution $F(q, n-k)$ that the F statistic follows):

$$F = \frac{\boxed{(\hat{\alpha}_1 + \hat{\alpha}_2 - 1)^2}}{\boxed{\widehat{Var}(\hat{\alpha}_1 + \hat{\alpha}_2)}} \sim F(\underset{q}{\boxed{1}}, \underset{n-k}{\boxed{24}})$$

generally, q is the number of restrictions under H_0.

We then need to compute both $\widehat{\sigma}^2$ and $\widehat{\text{Var}}(\cdot)$, where

$$\widehat{\sigma}^2 = \frac{\widehat{u}'\widehat{u}}{n-k} \approx \boxed{0.035}$$

Then, $\widehat{\text{Var}}(\widehat{\alpha}) = \widehat{\sigma}^2(X'X)^{-1}$, where $(X'X)^{-1} \approx$:

$$\begin{array}{c c} & \begin{array}{ccc} \hat{\alpha}_0 & \hat{\alpha}_1 & \hat{\alpha}_2 \end{array} \\ \begin{array}{c} \hat{\alpha}_0 \\ \hat{\alpha}_1 \\ \hat{\alpha}_2 \end{array} & \begin{pmatrix} \boxed{3.01} & \boxed{-0.56} & \boxed{0.034} \\ \boxed{-0.56} & \boxed{0.45} & \boxed{-0.27} \\ \boxed{0.034} & \boxed{-0.27} & \boxed{0.21} \end{pmatrix} \end{array}$$

Note that $\widehat{\text{Var}}(\widehat{\alpha}_1 + \widehat{\alpha}_2) = \widehat{\text{Var}}(\widehat{\alpha}_1) + \widehat{\text{Var}}(\widehat{\alpha}_2) + 2\widehat{\text{Cov}}(\widehat{\alpha}_1, \widehat{\alpha}_2)$. Then (substitute in the computed values, and simplify):

$$\widehat{\text{Var}}(\widehat{\alpha}_1 + \widehat{\alpha}_2) = \widehat{\sigma}^2(\underbrace{\boxed{[\ 0.45\ ]}}_{\widehat{Var}(\widehat{\alpha}_1)} + \underbrace{\boxed{[\ 0.21\ ]}}_{\widehat{Var}(\widehat{\alpha}_2)} + \underbrace{\boxed{[2(-0.27)]}}_{2\widehat{Cov}(\widehat{\alpha}_1,\widehat{\alpha}_2)}) \approx \boxed{[0.0039\ ]}$$

So (compute the F statistic, and compare it to the appropriate critical value):

i.e. do not reject constant retns. to scale at 95% level

$$F \approx \boxed{[\ 0.12\ ]}\ \cancel{>}\ /\ < F_{\boxed{[\ 0.95\ ]}}(\boxed{[\ \ 1\ \ ]}, \boxed{[\ 24\ ]}) \approx \boxed{[\ 4.26\ ]}$$

note: Prob = 0.7366 > 0.05

and so we ~~reject~~ / do not reject the null at the 95% level.

4. A generalization of the Cobb-Douglas model is the **translog model (eq02)**:

$$\ln(Q_i) = \underbrace{\alpha_0 + \alpha_1 \ln(L_i) + \alpha_2 \ln(K_i)}_{\text{Cobb-Douglas model}} + \frac{1}{2}\alpha_3(\ln(L_i))^2 + \frac{1}{2}\alpha_4(\ln(K_i))^2 + \alpha_5 \ln(L_i)\ln(K_i) + u_i,$$

q=3 restriction under H_0

which reduces to the Cobb-Douglas model under $H_0 : \alpha_3 = \alpha_4 = \alpha_5 = 0$. Run the translog regression:

$$\begin{aligned}\widehat{\ln(Q_i)} &= \boxed{[\ 0.94\ ]} + \boxed{[\ 3.61\ ]}\ln(L_i) + \boxed{[\ -1.89\ ]}\ln(K_i) \\ &+ \boxed{[-0.48\ ]}(\ln(L_i))^2 + \boxed{[\ 0.043\ ]}(\ln(K_i))^2 + \boxed{[\ 0.31\ ]}\ln(L_i)\ln(K_i)\end{aligned}$$

EViews gives the "Wald" F statistic as:

i.e. translog model reduces to Cobb-Douglas at 95% level.

$k=6$

$$F \approx \boxed{[\ 1.77\ ]}\ \cancel{>}\ < F_{\boxed{[\ 0.95\ ]}}(\overset{q}{\boxed{[\ \ 3\ \ ]}}, \overset{n-k}{\boxed{[\ 21\ ]}}) \approx \boxed{[\ 3.07\ ]}$$

and so we ~~reject~~ / do not reject the null hypothesis at the 95% level.

note: Prob = 0.1841 > 0.05

5. What is the estimated **capital elasticity of output** from the *translog* model?

$$\widehat{\eta} := \frac{\widehat{\partial \ln(Q_i)}}{\partial \ln(K_i)} \approx \boxed{[-1.89\ ]} + \boxed{[2(0.043)]}\ln(K_i) + \boxed{[\ 0.31\ ]}\ln(L_i).$$

Replace $\ln(L_i)$ and $\ln(K_i)$ by their sample means $\frac{1}{n}\sum_{i=1}^{n}\ln(L_i) \approx \boxed{[\ 5.76\ ]}$ and $\frac{1}{n}\sum_{i=1}^{n}\ln(K_i) \approx \boxed{[\ 7.45\ ]}$, to give the capital elasticity of output evaluated at the sample means:

$$\widehat{\eta} \text{ at sample means } \approx \boxed{[\ 0.54\ ]}$$

so that a $\boxed{[\ 10\ ]}$% increase/~~decrease~~ in capital results in a $\boxed{[\ 5.4\ ]}$% increase~~/decrease~~ in output.

compare eq01: capital elasticity of output 3.8% (if K ↑ 10%)

also consider $\bar{R}^2$: eq01 (93.9%) eq02 (94.4%),

this suggests that eq02 fits better than eq01;

however, part 4 shows that the translog reduces

to the Cobb-Douglas, and so eq01 is preferable!